

Optical quantum memory in atomic ensembles: physical principles, experiments, and potential of application in a quantum repeater

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Abstract. We outline the physical principles of optical quantum memory in atomic ensembles, with the emphasis on protocols that have been successfully implemented experimentally. Among the protocols under consideration, prominence is given to the regularities of quantum memory on photon echo and slow light and to their implementations in crystals doped with rare-earth ions, which have a long quantum coherence lifetime on optical and spin transitions. We analyze the approaches and problems in achieving high efficiency and other practically significant parameters of quantum memory in real experimental conditions. Also discussed are ways to further improve optical quantum memory and the potential of its application in developing an optical quantum repeater.

Keywords: optical quantum memory, two- and three-level atomic ensembles, optical and spin coherence, electromagnetically induced transparency, off-resonant Raman transitions, Autler–Townes splitting, photon echo, crystals with rare-earth ions, quantum repeaters

1. Introduction

Quantum memory (QM) is a device that allows storing, for a certain period of time, quantum information, which is defined by the state of qubits (quantum bits) [1]. The quantum state of a qubit is a superposition of two basis states $|0\rangle$ and $|1\rangle$:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where α and β are the probability amplitudes of these states that satisfy the condition $|\alpha|^2 + |\beta|^2 = 1$. An arbitrary state of a qubit corresponds to a point on the surface of a unit sphere defined by the zenith θ and azimuth φ angles, and the probability amplitudes are given in the form $\alpha = \cos(\theta/2)$, $\beta = \exp(i\varphi)\sin(\theta/2)$. A set of several qubits can be in an entangled quantum state, which is the most important resource of quantum information science that has no counterpart in classical physics, but such states can no longer be represented by a visual classical model.

In optical QM, the carriers of quantum states are often single-photon qubits, which, when stored in a QM cell, are transferred to the quantum states of stationary qubits (see review Refs [2–4]). The states of photonic qubits are encoded into various degrees of freedom of the photon, such as polarization [5], frequency [6], spatial [7] or temporal [8] modes, orbital angular momentum [9, 10], and phase-time encoding [11]. Different physical systems are used as the

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quantum memory cell, such as cold or warm atomic gas [7, 12], quantum dots [13, 14], single atoms in a high-Q cavity [15, 16], cavity configurations [17], point defects in diamonds [18–20], and crystals doped with rare-earth ions (REIs) [21–24]. Each of these systems, with its own advantages and disadvantages, may be the most interesting for solving a particular practical problem. In this review, we focus only on the versions (protocols) of QM based on atomic ensembles that allow storing a large number of photonic qubits in one QM cell, in which significant experimental results have been achieved, and describe the physical principles of their operation. The emphasis will be placed on QM in crystals doped with REIs [25, 26], which have a long lifetime of spin and optical coherence, making them candidates for storing quantum states of optical and microwave photons.

QM is necessary for the development of many quantum technologies. For example, it is a basic element for the implementation of an optical quantum repeater [27]. It is well known that the longest range of photonic qubit transmission in standard fiber-optic lines is about 100 km [28] and becomes hardly possible for a distance of 500 km. A quantum repeater allows solving the problem of transmitting the photon quantum state over long distances. A quantum repeater [29] implements the exchange of quantum entanglement using quantum teleportation between adjacent segments (see review Refs [30–33]), where the length of each segment is determined by signal attenuation in this section. QM is a key element of such a repeater, since it allows maintaining the quantum state of entangled photons in different nodes of the quantum network until quantum entanglement is established between the extreme nodes of the network. In this case, the procedure for establishing entanglement at the ends of each individual segment can be carried out independently.

It is significant that in addition to its use in a quantum repeater, QM has other important applications. For example, QM devices are necessary for the development of a universal quantum computer [34] and are important for studying fundamental issues of quantum mechanics [35, 36] and quantum metrology and magnetometry [37], and for use as a source [38, 39] and detector [40] of single-photon light fields. The latest advances in the field of the above applications are discussed at length in review Refs [3, 4]. For each specific application, different basic parameters and criteria may matter for the operation of the QM, which will be discussed below.

2. Basic parameters of optical quantum memory

Let us consider the main critical parameters that characterize the operation of the QM and distinguish it from classical memory. We will also dwell on the best experimental values of these parameters achieved in solid-state systems doped with REIs.

Efficiency. By efficiency is meant the ratio of the average number of photons at the output of the QM cell to the average number of signal photons sent to its input. Ideally, it should be equal to 100%. At present, the best efficiency of a solid-state optical QM is 69%, obtained in the photon echo protocol on a $\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$ crystal [41]. A slightly higher value of 76% was achieved in the classical memory mode in the same crystal using the electromagnetically induced transparency (EIT) protocol [42]. The efficiency of QM is quite easily measured in an experiment, but it is important that the reconstructed signal be as close as possible to the quantum state of the signal pulse supplied to its input. To

characterize this quality of the QM, a criterion of fidelity (accuracy) of the QM operation is introduced.

Fidelity is a measure of the accuracy of the reconstruction of the quantum state of signal radiation. In the general case, the quantum state of the signal and reconstructed fields is described by their density matrices ρ and ρ' , respectively, for which the fidelity F is expressed by the formula [1, 43]

$$F = \left(\text{Tr} \sqrt{\sqrt{\rho} \rho' \sqrt{\rho}} \right)^2,$$

its value lying in the range $0 \leq F \leq 1$, from orthogonal to identical quantum states. There are error correction methods [44] that allow reducing the requirements for the parameters of the applied QM. There is also a threshold of fidelity F , below which the advantages of QM over classical memory do not manifest themselves. In the case of discrete variables [1], the threshold is $F \geq 2/3$ [45], while for continuous variables [46] it is necessary to have $F \geq 1/2$ [47]. In Ref. [5], the best QM fidelity of 99.9% was achieved by storing a photon state polarization qubits in an $\text{Nd}^{3+}:\text{YVO}_4$ crystal.

Storage time and on-demand reading of stored light states. For QM, one of the important performance criteria is the storage time. Specifically, an optical QM in a quantum repeater must exhibit a storage time of at least 1 ms to produce entanglement between quantum network nodes located 200 km apart. Increasing the distance between nodes calls for a significantly longer QM lifetime. In Ref. [48], the authors demonstrated the feasibility of lengthening the QM lifetime to 6 hours using transitions between hyperfine sublevels of the ground state of europium ions in a $^{151}\text{Eu}^{3+}:\text{Y}_2\text{SiO}_5$ crystal at a temperature of 2 K. To make such a QM, it is necessary to develop efficient techniques for transferring quantum information from the signal light field to the quantum coherence of hyperfine transitions of europium ions.

Using the QM protocol reliant on the EIT effect in the $\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$ crystal, the authors of Ref. [49] have already demonstrated a QM lifetime of 1 min, though with insufficiently high efficiency and signal-to-noise ratio. In this case, the increase in the lifetime of the QM in spin transitions was achieved using additional radio-frequency (RF) pulses. The possibility of choosing an arbitrary point in time in reading the recorded information from the QM cell is another of its important properties; without it, the QM will be an ordinary uncontrollable delay line.

Multimode capacity determines the number of photonic qubits that can be stored in a QM cell. To a large extent, the capacity depends on the chosen QM protocol. In Ref. [8], the storage of 1060 temporal modes in a $\text{Tm}^{3+}:\text{YAG}$ crystal was experimentally demonstrated using the photon echo QM protocol. In Ref. [29], it was shown that storing N optical modes (temporal, spectral, or spatial) in a QM cell enables the multiplexing procedure and thereby increases N -fold the rate of establishing quantum entanglement between remote nodes of optical quantum communications. Multiplexing involves the use of address writing and a read-out of photonic qubits [50].

Operating wavelength and bandwidth. Different QM applications can use different carrier frequencies of photonic qubits. For example, for a quantum repeater to operate by existing fiber-optic communication lines, the QM must operate at telecommunication wavelengths in the 1550-nm region [28]. For a QM operating at other wavelengths, it is

necessary to develop a quantum frequency converter [51]. To increase the throughput and shorten the operation durations, it is important to use shorter light pulses [52, 53]. For example, in Ref. [54], it was shown that the operating spectral width of a QM may range up to 50 GHz.

Integrability. For the success of practical applications, in demand are miniature, down to micro- and nanosized QM devices capable of integration into existing and developing devices. This line has been rapidly developing in the last decade (see review Ref. [55]): QM in an optical fiber doped with Er^{3+} ions has been demonstrated [56], waveguide structures in crystals activated by REIs are under development [57, 58], and QM in nanosized cavities based on mesoscopic REI ensembles is being developed [54].

Despite some success and basic results demonstrating the feasibility of making a compact QM, serious problems remain to be solved, which do not allow achieving the main parameters of the QM in one device that are critical for practical application. For instance, it is noteworthy that the achieved efficiency (end-to-end efficiency) of compact QM devices is only ten percent, which is worse than the results obtained earlier for solid-state QM and calls for improving integrated QM devices and searching for more advanced QM protocols.

The use of solid-state QM carriers is of great importance for the development of a chip-based QM integrable into different compact devices. The ultimate values of efficiency, storage time, and capacity of QM in REI-doped crystals largely depend on the parameters of both the crystals themselves and the spectroscopic parameters of the REIs embedded in them. At the same time, the basic parameters of the QM and the magnitude of parasitic noise are greatly influenced by the quality of the implementation of the QM protocols in use, whose improvement currently remains a serious problem. By the protocol is meant a method for transferring photonic qubits to stationary QM carriers and storing them for the required time, with their subsequent conversion into an emitted photonic qubit. In the past two decades, a number of QM protocols have been proposed that continue to be developed, while great hopes are also being pinned on the search for the most suitable substances, their spectroscopic parameters, and methods for controlling quantum coherence and suppressing decoherence effects.

3. Quantum memory in atomic ensembles

Coherent atomic ensembles are of great interest as stationary carriers of quantum information for two important reasons. First of all, they readily ensure a sufficiently large constant of interaction with photons, which provides a high probability of storing a photon in the system of atoms. Second, atomic ensembles enable storing a large number of photonic qubits in one QM cell. At the same time, the use of a large number of atoms leads to the emergence of a number of coherent and nonlinear effects in the interaction of light with the atomic ensemble [59–61], which may either play a large positive role or cause negative effects in the implementation of the QM in an ensemble of atoms.

In the development of optical QM in resonant atomic ensembles, much attention has been paid to the use of such coherent effects as EIT and photon echo, which make it possible to control the propagation of light in a medium, its interaction with atoms, and the behavior of atomic coherence. At the same time, the emerging coherent and multi-

particle effects in the behavior of an atomic ensemble may greatly complicate the implementation of QM, which calls for the implementation of the most accurate reversibility in the light–atom interaction. For instance, the well-known coherent Dicke superradiance effect in the generation of a short light pulse by an atomic ensemble [62–67] provides highly efficient energy transfer from atoms to radiation. Such behavior of atoms seems valuable for the implementation of efficient quantum information readout from atoms to radiation. However, the spontaneity and irreversibility of this effect are undesirable for optical QM, which should provide coherent control of reversible behavior in the interaction of photons with an atomic ensemble without entailing quantum noise.

Significantly enhanced interaction of photons with an ensemble of atoms facilitates the implementation of the photon–atom interface at the stages of recording and reading quantum information, which are carried out at short times of the QM operation. However, at long times, weaker incoherent interactions exert a strong negative effect, among which magnetic spin-spin interactions play a major role. The development and application of methods for suppressing the influence of these interactions became one of the most important tasks in the implementation of long-lived QM [68, 69].

Below, we consider the properties of the basic protocols of ensemble QM, whose intensive theoretical and experimental developments belong to two main groups. The first includes the QM protocol based on the EIT effect [70–72], the protocol based on off-resonant Raman interaction [73, 74], and the dynamic splitting of the resonance line due to the Autler-Townes effect (ATS protocol) [75–77]. The second group of protocols involves the use of highly reversible photon echo schemes [2, 26, 78], which provide reversible dynamics of atoms and the light interacting with them.

The theoretical description of different optical QM protocols is often based on the Heisenberg–Langevin equations for slowly varying field $\hat{a}(z, t)$ and atomic $(\sigma_-^j(t), \sigma_+^j(t), \sigma_z^j(t))$ operators. In the case of interaction of light with an ensemble of two-level atoms, they are often reduced to a system of equations commonly known as the Maxwell–Bloch (MB) quantum equations [59, 61], which were first introduced by Tito Arecchi and Rodolfo Banifacio [79]. In this connection, it was proposed in [80] to call these equations the Maxwell–Bloch and Arecchi–Banifacio (MB–AB) equations. Below, we write these equations with the inclusion of the interaction of atoms with a field mode with different field amplitudes in the cross section of the light beam, which is typical of optical (single-mode) waveguides:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) \hat{a}(z, t) &= i \frac{1}{2} \sum_{j=1}^N g^*(\mathbf{r}_\perp^j) \sigma_-^j(z_j, t) \delta(z - z_j), \\ \frac{\partial}{\partial t} \sigma_-^j(t) &= - \left(\frac{\gamma}{2} + i\Delta_j \right) \sigma_-^j(t) - i \frac{g(\mathbf{r}_\perp^j)}{2} \hat{a}(z_j, t) \sigma_3^j(t) + \sqrt{\gamma} \hat{F}_j(t), \end{aligned} \quad (1)$$

$$\frac{\partial}{\partial t} \sigma_3^j(t) = -i \frac{\Omega}{2} [\hat{a}^+(z_j, t) \sigma_-^j(t) - \sigma_+^j(t) \hat{a}(z_j, t)],$$

where $\sigma_\pm^j = 1/2(\sigma_1^j \pm \sigma_2^j)$, $(\sigma_+^j = (\sigma_-^j)^\dagger)$, $\sigma_{1,2,3}^j$ are the isospin operators of the j th atom, $(\sigma_+^j|1_j\rangle = |2_j\rangle, \sigma_-^j|2_j\rangle = |1_j\rangle, \sigma_3^j|1_j\rangle = -|1_j\rangle, \sigma_3^j|2_j\rangle = |2_j\rangle)$, the field operator $\hat{a}(z, t) = (1/\sqrt{2\pi}) \int d\mathbf{k} \hat{a}_{\mathbf{k}}(t) \exp[i(\mathbf{k} - \mathbf{k}_0)z]$, $[\hat{a}(z', t), \hat{a}^\dagger(z, t)] =$

$\delta(z-z')$, $\gamma = 2/T_2$ is the relaxation constant of atomic coherence, and its corresponding Langevin forces $\sqrt{\gamma}\hat{F}_j(t)$ and $\sqrt{\gamma}\hat{F}_j^\dagger(t)$, whose properties are determined by incoherent interactions of atoms with reservoir modes, are often defined by the correlators $\langle \hat{F}_j(t)\hat{F}_j^\dagger(t') \rangle = \delta(t-t')$, which are the same for all atoms [61, 81], $g(\mathbf{r}_\perp) = E_0 f(\mathbf{r}_\perp) \langle \mathbf{d} \mathbf{e} \rangle / \hbar$ is the single-photon Rabi frequency of the atom with coordinate $\mathbf{r}_\perp$, $\mathbf{d}$ is the dipole matrix element of the atomic transition, and the electric field of the light wave (signal and reconstructed) is defined by the operator

$$\mathbf{E}(\mathbf{r}, t) = iE_0 \mathbf{e} f(\mathbf{r}_\perp) a(z, t) \exp[-i(\omega_0 t - k_0 z)] + \text{c.c.}, \quad (2)$$

where $E_0 = (\hbar\omega_0/2\epsilon_0\epsilon(\omega_0)S)^{1/2}$, ω_0 and $\mathbf{k}_0 \uparrow \mathbf{z}$ are the carrier frequency and wave vector of radiation propagating in a light beam with cross section $S = \pi a^2$, $v_g = \partial\omega_0/\partial k_0$, ϵ_0 and $\epsilon(\omega_0)$ are the permittivity of vacuum and medium, $\mathbf{e}$ is the polarization vector of the photon electric field, the transverse structure of the electric field is described by the membrane function $f(\mathbf{r}_\perp)$, which is often specified by the Gaussian function $f(\mathbf{r}_\perp) = \exp\{-(x^2 + y^2)/2a^2\}$ in a single-mode waveguide, where a is the working radius of the fiber [82]. Putting $f(\mathbf{r}_\perp) = f(0) = 1$ in Eqns (1), we can proceed to consider the case of a plane light field, which is often considered theoretically [59]. For simplicity, the MB–AB equations do not include the influence of relaxation processes in the optical channel, which may be significant when using long optical waveguides and may also manifest itself in the integral waveguide structures being developed [83].

The MB–AB system of equations is solved under different approximations in the description of a broad range of problems, including laser generation [79, 81, 84], self-induced transparency [85–88], Dicke superradiance [62–66], and photon echo [86, 89–97]. The search for exact analytical solutions of the nonlinear quantum system of MB–AB Eqns (1) is a complex problem. Ignoring atomic relaxation (i.e., for $\gamma t \ll 1$), Eqns (1) correspond to the Tavis–Cummings Hamiltonian [98]. In the case of inhomogeneous broadening of the resonance atomic transition, the MB–AB Eqns (1) are also known as the inhomogeneous Dicke model, for which a number of exact solutions have been obtained in the description of Dicke superradiance using the quantum Bethe ansatz method and the *quantum inverse scattering method* (QISM) [67, 99, 100]. In the classical description of the field, G.L. Lamb [86] and Ablowitz [87] had earlier shown that the solution of nonlinear Eqns (1) is reduced to the solution of the linear integro-differential Gelfand–Levitan–Marchenko equation when use is made of the ISP. However, analytical solutions of this equation can be found only for solitons arising in the depth of the medium as a result of the interaction of the input pulse with atoms. In this case, at the input to the medium, the behavior of radiation and atoms undergoes a complex transition stage, where the formation of solitons is combined with nonstationary oscillations of atomic coherence. Taking into account the physical conditions of a real experiment, the arbitrary initial conditions of the medium, and its multiple pulse excitation further complicates the application of the ISP [101], which invites further development of this method [102–104].

Depending on the characteristics of the arising problems, the system of MB–AB Eqns (1) $\sigma_{-}^j(t)$ can be refined by the introduction of additional interactions, atomic energy levels, the specific nature of the inhomogeneous broadening of resonance lines and the characteristics of atomic relaxation,

the use of additional controlling laser and RF fields, the use of waveguides, cavities, etc. The wide range of problems to be solved makes relevant the development of other analytical and numerical approaches to solve the MB–AB equations (1), which make it possible to obtain solutions with an accuracy sufficient to describe existing experiments (see Refs [105–109] and references therein).

MB–AB Eqns (1) are also a basic theoretical model for describing QM protocols, which is supplemented and modified with the inclusion of specific features of a real experiment. The description of specific QM protocols is often focused on the use of quantum light states containing a small number of photons, but light fields with a large number of photons [105] and solitons, whose quantum states are of growing interest [110–115], are also considered to be quantum information carriers. In this connection, of importance is the universality of the behavior of the ensemble-based QM efficiency, so that it is sufficient to consider light fields classically to study it. However, analyzing the accuracy in the operation of QM with quantum light states invites the use of a rigorous quantum approach that takes into account the influence of quantum fluctuations. The tasks of QM are focused on light–atoms interactions, whereby it is important to ensure complete (reversible) absorption of signal light pulses by atoms without soliton production at the output of the medium.

After absorption of signal pulses, atoms are often additionally exposed to intense control laser pulses, whose use makes it possible to control the quantum coherence of the atoms and reversibly transfer it to long-lived QM levels at appropriate points in time. Due to the high intensity of pulses and their nonlinear interaction with atoms, controlling their parameters in the QM cell becomes a difficult task.

4. Quantum memory protocols for three-level atomic ensembles

Much attention is currently being directed to the development of QM protocols based on the EIT effect and off-resonant Raman scattering (ORS), and interest has recently increased in the QM protocol involving the ATS effect. These QM versions are described by system of equations (1) extended to a three-level atomic model with the additional inclusion of the effect of an intense control field characterized by the Rabi frequency $\Omega_c(\mathbf{r}^j, t)$, which operates by the adjacent atomic transition $|2\rangle \leftrightarrow |3\rangle$ (Fig. 1). Assuming a weak signal field and taking into account the response of atoms in the linear approximation (disregarding the weak atomic coherence at the $|2\rangle \leftrightarrow |3\rangle$ transition), the system under study can be described by the Heisenberg–Langevin equations for slow variables of the signal field and atoms [61]:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) \hat{a}(z, t) &= i \frac{1}{2} \sum_{j=1}^N g^*(\mathbf{r}_\perp^j) \sigma_{12}^j(z, t) \delta(z - z_j), \\ \frac{\partial}{\partial t} \sigma_{12}^j(t) &= - \left(\frac{1}{2} \gamma_{12} + iA_j \right) \sigma_{12}^j(t) + i \frac{g(\mathbf{r}_\perp^j)}{2} \hat{a}(z_j, t) \\ &\quad + i\Omega_c(\mathbf{r}^j, t) \sigma_{13}^j(t) + \sqrt{\gamma_{13}} \hat{F}_{12}^j(t), \\ \frac{\partial}{\partial t} \sigma_{13}^j(t) &= - \left(\frac{1}{2A_j} \gamma_{13} + i\delta_j \right) \sigma_{12}^j(t) + i\Omega_c^*(\mathbf{r}^j, t) \sigma_{12}^j(t) \\ &\quad + \sqrt{\gamma_{13}} \hat{F}_{12}^j(t), \end{aligned} \quad (3)$$

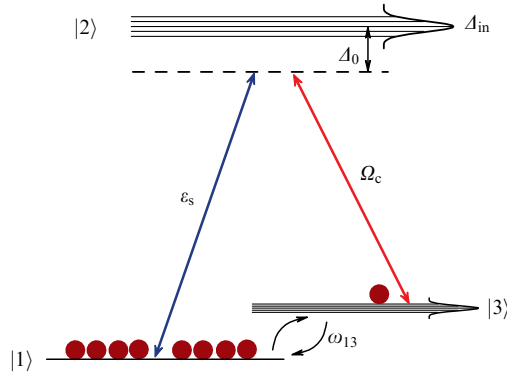


Figure 1. Three-level system for implementing OFS protocol; EIT and ATS are implemented with zero frequency offset $\Delta_0 = 0$.

where $\delta_j = [\omega_{31}^j - (\omega_s - \omega_c)]$ is the Raman resonance detuning; $\Delta_j = \omega_{21}^j - \omega_s$ is the frequency detuning at the optical transition; ω_s and ω_c are the frequencies of the signal and control fields; ω_{nm}^j are the frequencies of the j th atom at the transition $|m_j\rangle \leftrightarrow |n_j\rangle$; and it is taken into account that, for a weak signal pulse, $\langle \sigma_z^j \rangle = \langle [\sigma_{22}(t) - \sigma_{11}^j(t)] \rangle \cong -\langle \sigma_{11}^j(t) \rangle = -1$ and $\sigma_z^j \cong -1$ and that the nonzero correlators of the Langevin forces $\langle \hat{F}_{12}^j(t) \hat{F}_{21}^j(t') \rangle = \langle \hat{F}_{13}^j(t) \hat{F}_{31}^j(t') \rangle = \delta(t - t')$. Also taken into account in Eqns (3) is the possibility of describing the interaction of light fields with atoms in a single-mode waveguide, which is often used in the optical QM under development [55]. In this case, a dependence appears in the Rabi frequency of the interaction of an atom with the signal $g(\mathbf{r}_\perp^j)$ and control fields $\Omega_c(\mathbf{r}^j, t)$ on the arrangement of the atoms in the cross section of the waveguide.

The propagation of the signal field and the efficiency of its storage in the system of atoms may be examined by studying the behavior of the average values for the atomic coherence and field. The Langevin forces do not contribute to their behavior, since their average values are zero $\langle \hat{F}_{nm}^j(t) \rangle = 0$, and Eqns (3) are reduced to the MB–AB equations for a three-level system of atoms that do not contain Langevin forces. However, in the case of a more complex level structure, for example, including hyperfine and Zeeman sublevels in the ground and optically excited states, the influence of these forces may be significant [116].

The QM protocols under consideration (EIT, ORS, and ATS) differ in the value of the detuning Δ_j and the Rabi frequency $\Omega_c(\mathbf{r}^j, t)$ in Eqns (3). The main properties of these protocols were adequately studied in the plane wave approximation $\Omega_c(\mathbf{r}^j, t) = \Omega_c(t)$ in Refs [70, 75–77, 117–121]. In real systems, as a rule, the optical transition experiences inhomogeneous broadening; however, in the EIT, ORS, and ATS protocols, its influence is not used in light control, and it can be largely weakened [122] and taken into account by introducing an effective atomic-coherence decay constant (see below).

4.1 Electromagnetically induced transparency protocol

The EIT protocol is implemented for weak signal fields in a narrow transparency window of the optical transition $|1\rangle \leftrightarrow |2\rangle$, which occurs in the atomic-transition Λ -scheme under consideration in the presence of a control field Ω_c at the adjacent transition $|2\rangle \leftrightarrow |3\rangle$. The resultant dip in the absorption spectrum of the $|1\rangle \leftrightarrow |2\rangle$ transition is accompanied by a sharp spectral susceptibility, which, according to

Eqns (3), has the well-known form [61, 123, 124]

$$\chi(\Delta) \sim \frac{g(\gamma_{13} + i\Delta)}{(\gamma_{13} + i\Delta)(\gamma_{12} + i\Delta) + |\Omega_c|^2}.$$

This effect is caused by the Autler–Townes splitting of the upper level 3 into two sublevels under the action of the control field Ω_c [125] and under conditions of the action of destructive (Fano) interference of the arising two states on the operation of the signal pulse. Fano interference is realized under a weak control field $\Omega_c < \gamma_{12}$, determining the characteristic shape of the narrow spectral dip, and is accompanied by the appearance of the so-called dark state on the atomic transition $|1\rangle \leftrightarrow |3\rangle$ under conditions of coherent population trapping [160, 161]. Steep spectral dispersion $\chi(\Delta)$ in the absence of absorption results in the EIT effect, which underlies the EIT protocol of QM [127].

The influence of the inhomogeneous broadening caused by the Doppler effect on the shape of the dip in the absorption spectrum was investigated in Ref. [128]. It was shown that the negative influence of the Doppler effect on the EIT protocol is strongly suppressed in the case of collinear geometry of the propagation of the signal and control radiation and a small difference in the optical frequencies of the atomic transitions. At the same time, as shown in Refs [129, 130], in warm gas cells, the influence of the inhomogeneous broadening of optical transitions can negatively affect the parameters of the narrow dip and the reliability of the EIT protocol of the QM due to atomic velocity-dependent changes in light–atom interactions.

In the region of a narrow absorption dip, a strong slowing down occurs of the group velocity v_{eit} of the signal field $\hat{a}(t - z_j/v_{\text{eit}})$, which drops sharply with increasing optical thickness and with decreasing amplitude of the control field $v_{\text{eit}} = v_g/(1 + Ng^2/|\Omega_c|^2)$. Such light propagation is accompanied by the formation of ‘dark state’ polaritons [117, 131, 132], the main part of whose energy is contained in the macroscopic spin coherence $\sigma_{13}(t, z) = -(g/\Omega_c) \hat{a}(t - z/v_{\text{eit}})$, where $\sigma_{13}(t, z) = \sum_{j=1}^N \sigma_{13}^j(t, z_j) \delta(z - z_j)$. The optical component of the polariton becomes greatly weakened, $\sigma_{12}(t, z) = -i(1/\Omega_c)(\partial/\partial t) \hat{a}(t - z/v_{\text{eit}})$, with a decrease in the group velocity in an optically thick medium ($Ng^2 \gg |\Omega_c|^2$), which may become many orders of magnitude lower than the speed of light. In this case, the spatial size of the polariton pulse becomes smaller than the linear dimensions of the atomic medium ($v_{\text{eit}} \delta t_s < L$, where δt_s is the pulse duration of the input light signal).

The perfect storage of the quantum state of the polariton is realized in the adiabatically slow switching off of the control field $\Omega_c(t) \rightarrow 0$, when the light component of the polariton is completely transferred to the spin coherence $\sigma_{13}(z)$, which is accompanied by the stopping of the polariton [70, 117]. Fast (instantaneous) switching off of the control field does not introduce significant loss for a large initial fraction of spin coherence in the polariton, which occurs in an optically thick resonant medium [118], and therefore such regulation of the control light came to be often used in the EIT protocol [133, 134]. A detailed analysis [74] showed that the highest efficiency of the EIT protocol is achieved when using a temporal shape of turning on/off the control field that is optimal for the temporal form of the signal pulse, as is the case for the QM in an atom in a cavity. Furthermore, the highest efficiency is achieved by restoring the signal pulse in the direction opposite to its initial

Table 1. Characteristics of QM based on EIT protocol for ensembles of atomic gases (w — warm gas, c — cold gas, laser cooling, WCS — weak coherent state, n — average number of photons per pulse).

Gas type (w/c)	λ , nm	QM efficiency (η)	Storage time τ	QM lifetime	Single-photon level	Spectral width	Fidelity	References
w — ^{87}Rb	795	67	100 ns	1.2 μs	(WCS) $n = 0.6$	~ 10 MHz	0.97	[136]
c — ^{133}Cs	895	69	4 μs	15 μs	(WCS) $n \geq 0.02$	~ 1 MHz	0.85 ($n = 0.1$) 0.97 ($n = 1$)	[139]
c — ^{133}Cs	895	87		15 μs	+	~ 3 MHz		[140]
w — ^{133}Cs	895	14 at $\tau = 0$		430 ms		~ 200 kHz		[137]
c — ^{133}Cs	895	92	1 μs	~ 325 μs		~ 5 MHz		[135]
w — ^{87}Rb	795	1.1 (End-to-End)	200 ns		+	350 MHz		[141]

propagation (in the backward geometry), which was first proposed for the CRIB QM protocol [78] (see below). The experiments conducted [135, 136] confirmed the higher efficiency of the EIT protocol in the reverse geometry. The authors of Ref. [118] called the reversed version of restoration a universal approach, which turned out to be more efficient for both the ATS and ORS protocol.

The best performance of the EIT protocol has been achieved experimentally in warm and laser-cooled atomic gases. An efficiency of 92% (for classical fields) and 67% (for single-photon fields) was demonstrated in Cs and Rb atomic ensembles [135, 136], where a record storage time (430 ms) was also achieved, though with a lower efficiency [137]. The basic parameters of the EIT protocol achieved in recent research are collected in Table 1. The main driver for the development of the EIT protocol in atomic gases is the possibility of operation at room temperature and lower cost compared to solids. However, the problem of the small spectral width of such QM (≤ 10 MHz) and the complexity of ensuring multiple-mode operation when using other degrees of freedom, including temporal and spatial, are yet to be solved.

Recently, a project appeared of a long-lived QM based on the EIT protocol in a warm gas cell containing two ensembles of atoms [138]. The first (A-) ensemble includes three-level atoms, where the signal pulse is stored in spin coherence. The second (B-) atomic ensemble is a gas of noble atoms, which exhibits a very long lifetime of spin coherence. After storing the signal pulse in the A-ensemble, the B-ensemble is introduced into a resonance exchange interaction with the A-ensemble via spin transitions of these ensembles by adjusting the magnitude of an external magnetic field. Therefore, an effective spin coherence transfer is ensured from the A-ensemble to the B-ensemble, where it can be stored for an hour or more after switching off the resonant exchange interaction. During readout, the sequence of actions is reproduced in the reverse order with the retrieval of the signal pulse in the EIT protocol. The possibility of using other protocols at the stages of recording and reading the signal pulse is of interest for studying other possibilities of using noble gases in optical QM.

The EIT protocol was also implemented in REI-doped crystals [142–149], including a $\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$ crystal, where storage of bright (classical) signal pulses was demonstrated over time intervals from seconds [150] to minutes [151]. Recently [152], the EIT protocol was implemented with an efficiency of 42% and a storage time of 2 μs for a coherent light pulse containing one photon on average. Using the dynamic decoupling technique, the storage time was lengthened to 1.28 s, but with a lower efficiency of 5.6%. Because of

the appearance of quantum noise, the signal pulse now contained an average of 10 photons.

The principles of implementing the EIT protocol have found numerous applications in different physical systems. Of great interest is the Duan–Lukin–Cirac–Zoller protocol (DLCZ protocol)—spontaneous heralded preparation of quantum entangled states [153], where one of the generated photons is restored from the atomic environment in the EIT protocol. The DLCZ protocol enjoys especially active use in optical quantum communications over long distances [154–156]. Methods to significantly reduce the group velocity of light under the conditions of the EIT effect were developed for the generation and control of so-called stationary light pulses. The possibility of the appearance of a stationary light field was first demonstrated using control fields in the form of a stationary standing light wave [157] and for backward propagating light fields with different frequencies [158, 159], causing a four-wave interaction of the components of the stationary light field. The last approach was actively developed [160] and came to be used in the implementation of quantum processing by single-photon fields [161].

Practical applications invite further improvement of the EIT QM protocol and its modifications. In its implementation in warm gases, one has to take into account the negative effect of atomic motion on spin coherence, the need to increase the frequency interval between spin sublevels to suppress quantum noise [162] arising from the manifestation of spontaneous four-wave mixing, and the importance of suppressing losses in the overall optical configuration [134]. However, four-wave mixing under conditions of the emergence of slow light may also be used to generate quantum light states [163].

4.2 Autler–Townes protocol

The ATS is also described by Eqns (3), though for a stronger control field $\Omega_c > \gamma_{12}$ [75–77], when the destructive interference effect is weakened and two separate lines caused by the Autler–Townes effect appear in the atomic system. The difference in the manifestation of EIT and ATS was recently demonstrated experimentally [164, 165]. Effective storage of the signal pulse is realized, as for the EIT protocol, in an optically thick medium and with the optimal choice of the magnitude and time shape of the control field $\Omega_c(t)$, but the ATS can already operate in the fast (nonadiabatic) QM mode. To protect atoms from their excessive excitation to optical levels, the pulse area of the control pulses is chosen equal to 2π . The best efficiency of the ATS was achieved in Ref. [77] for a 20-ns-long light pulse attenuated to a single-photon level and stored in the spin coherence of a rubidium Bose–Einstein condensate (BEC) at

a temperature of $T \approx 340$ nK. The efficiency of the ATS was 30% with a storage time of 15 μ s.

The feasibility of improving the efficiency of the ATS protocol is associated with an increase in the optical thickness [77], better optimization of the temporal shape of the control pulse, and the retrieval of the signal pulse in the inverted geometry [119], similar to the CRIB protocol [78]. In Ref. [166], the temporal scheme of the interaction of a signal pulse, involving two control π -pulses and the backward geometry of the signal retrieval [78], was applied to a homogeneously broadened system of $\approx 10^8$ ^{87}Rb rubidium atoms in a BEC cloud. Due to the absence of inhomogeneous broadening, this scheme was called the QM protocol reliant on the superradiance effect (SR protocol), which was earlier proposed in Ref. [167]. High efficiency was achieved by choosing the optimal optical thickness of the atomic transition and was $\alpha L \approx 9$ for a short signal light pulse of 10-ns duration with an exponentially increasing amplitude. It is pertinent to note that signal pulses of this temporal shape were previously used to implement microwave memory in a high-Q cavity mode with an efficiency of 90% [168].

Unfortunately, the control laser pulses in Ref. [166] were not short enough, which led to large losses of coherence due to incomplete suppression of spontaneous superradiance decay of atoms and reduced the efficiency of the protocol to 5%. Significant improvement in the efficiency of the ATS and SR protocols may call for substantial changes to the conditions of their implementation, for example, placing the atomic BEC in a cavity or using additional methods of suppression and recovery of quantum coherence of the atoms by controlling the Stark and Zeeman shifts of resonance lines in controlled external gradients of electric and magnetic fields.

4.3 Off-resonant Raman protocol

The theory of QM involving off-resonant Raman interaction (ORS protocol) was developed in Refs [74, 120, 121]. In this protocol, the frequency of the signal radiation is detuned from the line of the atomic optical transition by a value Δ , which significantly exceeds the line width Δ_{in} , so that the influence of the frequency detunings Δ_j of the atoms relative to the line center can be ignored. The nonresonant nature of the interaction of atoms at the optical transition suppresses luminescence and reduces quantum noise at the frequency of the signal radiation, which makes it possible to store short-duration light pulses. Such properties of the ORS protocol were demonstrated in storing single-photon field wave packets of nanosecond duration in a photonic crystal waveguide [169] with a signal-to-noise ratio of 2.6:1 and an efficiency of $27 \pm 1\%$.

In accordance with the general properties of the QM in atomic ensembles with the λ -scheme of atomic transitions [74], achieving high efficiency necessitates a high optical thickness. Since the intensity of the control field must be lower than the frequency detuning ($|Q_c|^2/\Delta^2 < 1$), more stringent requirements are imposed on the initial optical thickness of the atomic ensemble $\alpha L \gg \Delta^2/|Q_c|^2$. Satisfying this condition due to an increase in the intensity of the control field results in a stronger generation of spontaneously emitted noise photons by the four-wave mixing mechanism [162], as in the EIT protocol. A promising approach to suppress this noise is the use of an optical cavity tuned to resonance with the frequency of the signal radiation and to antiresonance with the frequencies of the noise photons [170, 171].

In recent paper [121], the ORS protocol was implemented for single-photon light pulses using the Ξ -configuration of optical transitions. An efficiency of $\sim 16.7\%$ and a storage time of 5 ns were achieved for a signal pulse with a record spectral width $\delta\omega_s \sim 1$ GHz at a frequency detuning $\Delta = 9$ GHz. More recently, using the backward geometry of signal pulse recovery [78], the authors of Ref. [172] were able to improve the ORS protocol efficiency to $65 \pm 6\%$ with a storage time of 1 μ s, exceeding for the first time the threshold of 50% required for the implementation of quantum memory for high-intensity light fields. The generalized implementation of the reversible dynamics proposed in Ref. [173] would also be expected to allow for efficient variation of the wavelength and duration of the recovered signal.

Resonant interaction of signal light fields with two- and three-level atomic ensembles can be used in the implementation of other versions of fast optical QM. The theoretical development of a number of fast QM versions, which are based on the use of various methods of controlling atomic coherence, is the concern of Refs [174–179]. The emergence of the first experiments will allow obtaining important information about their specific properties and the possibilities of quantum storing of short light fields.

5. Quantum memory protocols based on photon echo approach

Photon echo QM protocols harness the possibilities of restoring macroscopic coherence in inhomogeneously broadened atomic ensembles. This effect, called spin echo, was discovered by E. Hahn [180]. Thirteen years later, U.Kh. Kopvillem and V.R. Nagibarov demonstrated the possibility of implementing photon echo [181]. In Ref. [182], S.R. Hartmann and his colleagues independently predicted this effect and constructed a theory, which allowed it to be demonstrated experimentally for the first time. Below, we discuss the properties of a two-pulse (primary) photon echo in an optically thick medium and the difficulties of using it to store quantum states of light. We then move on to consider a time-reversible version of photon echo in an optically thick medium [78], which allows restoring the initial state of the signal field. Based on this approach, the subsequent sections will discuss the implementation of different optical QM protocols.

5.1 Photon echo

The formation of a two-pulse echo is shown in Fig. 2, where the first pulse, in which the transferred information is encoded by the temporal profile, is considered a signal pulse.

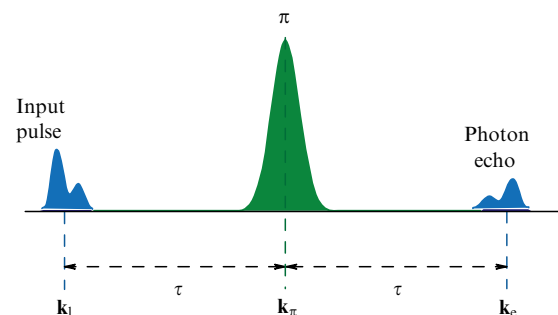


Figure 2. Sequence of light pulses involved in formation of primary photon echo.

This pulse excites macroscopic coherence, which vanishes due to the dephasing of atoms with different frequencies. The second intense rephasing pulse restores this coherence, and it leads to the appearance of a primary echo in the form of a time-reversed copy of the signal pulse (see Fig. 2) [183–185]. Such properties of the echo signal arise when the inhomogeneous broadening of the atomic transition is greater than the spectral width of the signal pulse $\Delta_{\text{in}} \gg \delta\omega_s$, as shown in Ref. [183] and borne out experimentally in Refs [184, 185]. A characteristic feature of the photon echo is the ability to propagate light pulses in different directions, which causes the primary echo emission in the direction of the wave vector $\mathbf{k}_e = 2\mathbf{k}_2 - \mathbf{k}_1$ (where $\mathbf{k}_{1,2}$ are the wave vectors of the exciting light pulses) and $\mathbf{k}_e \parallel \mathbf{k}_{1,2}$ for $\mathbf{k}_1 \parallel \mathbf{k}_2$. It was also discovered that the photon echo can reproduce, or acquire, the reversed wave front of signal pulses [186]. Photon echo allows one to obtain information about local interactions and relaxation processes of atoms in gases and solids, and therefore has become an effective method of optical spectroscopy [187]. Furthermore, the temporal and spatial properties of photon echo came to be used in problems of optical processing and storage of classical information [188–190].

In the case where the temporal shape of the echo signal and the signal pulse coincide, the efficiency of the photon echo is determined by the ratio of the echo signal intensity at the output of a medium of length L to the intensity of the signal (1st) pulse at its input $\eta = |\varepsilon_{\text{echo}}(L, t - 2\tau)|^2 / |\varepsilon_1(0, t)|^2$. In this case, it becomes possible to characterize different parameters of the echo signals by comparing the pulse areas of the signal $\theta_1(z)$ and echo $\theta_{\text{echo}}(z)$ (where $\theta_{1,\text{echo}}(z) = d/\hbar \int dt \varepsilon_{1,\text{echo}}(z, t)$ (z is the spatial coordinate in the atomic medium)). In Refs [93, 191], it was also discovered experimentally that the reproduction of the temporal behavior of the signal pulse and the primary echo is also possible in optically thick media, which makes it possible to express the efficiency of echo signal reconstruction in terms of the signal pulse areas: $\eta = |\theta_{\text{echo}}(L)|^2 / |\theta_1(0)|^2$. These properties of the pulse area of the echo signals make it possible to achieve significant progress in the theoretical description of optical QM using the equation for the pulse area of the photon echo signal, which was derived for the MB–AB system of equations in an optically thick medium [97]:

$$\frac{\partial}{\partial z} \theta_{\text{echo}}(z) = \frac{\alpha}{2} \left\{ 2P_e(z, 0) \cos^2 \frac{\theta_{\text{echo}}(z)}{2} + W_e(z, 0) \sin \theta_{\text{echo}}(z) \right\}, \quad (4)$$

where α is the resonance absorption coefficient of the medium [59], $P_e(z, 0)$ is the Fourier component of the phasing polarization at the carrier frequency, which is the source of the emitted echo signal, and $W_e(z, 0)$ is the slowly varying part of the atomic inversion also taken at the carrier frequency of the signal. From Eqn (4), we can find a solution for the pulse area of the primary echo signal [97]:

$$\theta_{\text{echo}}(z) = 2 \arctan \left[\exp \left(-\frac{2\tau}{T_2} \right) \sin \theta_1(0) \sin^2 \frac{\theta_2(z)}{2} \sinh \frac{\alpha z}{2} \right], \quad (5)$$

where we have for the pulse areas of the first pulse $\tan[\theta_1(z)/2] = \tan[\theta_1(0)/2] \exp(-\alpha z/2)$ and the second pulse $\sin^2(\theta_2(z)/2) = \gamma^2 / (\cosh^2(\beta - \alpha z/2) + \gamma^2)$ [85, 89], $\gamma = \tan[\theta_2(0)/2] / \sin \theta_1(0)$, $\beta = \ln \tan[\theta_1(0)/2]$. We restrict ourselves to the case of a weak input signal pulse ($\theta_1(0) \ll 1$)

and an echo signal $\theta_{\text{echo}}(z) \ll 1$ to obtain for its radiation efficiency

$$\eta = \frac{|\theta_{\text{echo}}(L)|^2}{|\theta_1(0)|^2} = 4 \exp \left(-\frac{4\tau}{T_2} \right) \sin^4 \frac{\theta_2(L)}{2} \sinh^2 \frac{\alpha L}{2}. \quad (6)$$

The highest η efficiency in expression (6) occurs for the pulse area $\theta_2(0) = \pi$, for which $\gamma \rightarrow \infty$ and $\sin^4(\theta_2(z)/2) = 1$ at all points of the medium ($\theta_2(z) \cong \pi$) and $\eta = 4 \exp(-4\tau/T_2) \sinh^2(\alpha L/2)$, which coincides with solution (6) obtained by solving the equation for the amplitude of the primary echo signal at $\theta_2(z) = \pi$. It is noteworthy that the maximum photon echo in an optically thick medium is achieved for $\theta_2(z) = \pi$, just as for the Hahn spin echo in a low-Q resonator [180] and for the photon echo in an optically thin medium [181, 182]. The maximum echo signal occurs due to the amplification of the echo during its propagation in an inverted optically thick medium. In this case, according to expressions (5) and (6), the amplification can make the echo signal more intense than the first exciting pulse ($|\theta_{\text{echo}}(L)| > |\theta_1(0)|$), which was later confirmed by numerical simulations [91] and borne out in experimental studies [93, 191, 192]. At the same time, according to expression (5), the amplification of the echo signal is limited, since its area cannot exceed π , which is typical of the propagation of light pulses in an inverted medium [85, 101, 193].

Although the primary echo at the output of the medium may be stronger than the signal pulse, the excitation caused by the signal pulse continues to persist in the atomic ensemble. Therefore, the effect of additional reading-out pulses allows one to read out information about the first pulse again, for example, in the signal of a three-pulse (stimulated) echo [93]. Using the area theorem, one can describe in detail the appearance of additional echo signals in an optically dense medium [95, 96]. However, the main disadvantage of using the primary echo in the QM is not the appearance of other echo signals, but its amplification during propagation in an inverted medium, which is accompanied by spontaneous emission of photons to produce quantum noise. This noise greatly reduces the accuracy of reconstruction of the quantum states of the signal fields [192, 194], making it impossible to use the primary two-pulse echo in quantum storage of the single-photon signal fields of the QM. Nor is it possible to use a three-pulse echo, which makes it necessary to develop other methods for generating photon echo signals that are not accompanied by the emergence of quantum noise.

The possibility of implementing optical QM with photon echo was shown in Ref. [78], where a protocol was proposed that provides the inversion of the dynamics of generation and interaction of the echo signal with atoms in comparison with the dynamics of absorption of the signal pulse in the same medium. Called the controlled reversible inhomogeneous broadening (CRIB protocol) [195–197], it then formed the basis for the development of other QM protocols that differ in the methods of implementing inhomogeneous line broadening and restoring atomic coherence in such an atomic system. Below, we consider the main properties of the CRIB protocol, following Refs [78, 198–201] and generalizations of their analysis [122, 173, 195, 202–205]. Next, we discuss the modifications of this protocol under development (GEM, AFC, ROSE, HYBRID), the features and advantages of their implementation in cavity schemes, the experimental results achieved, the current problems being solved, the problems of achieving practically significant parameters in

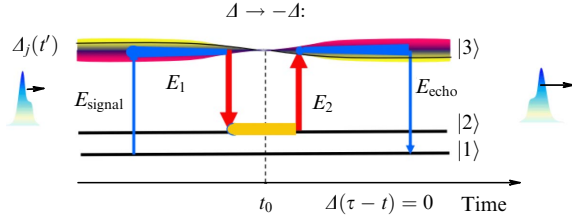


Figure 3. Temporal scheme of CRIB protocol implementation

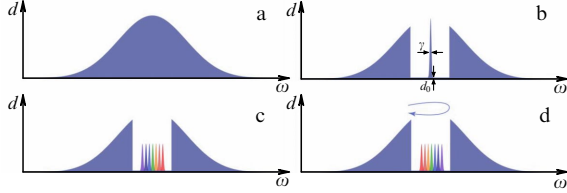


Figure 4. Spectral scheme of CRIB protocol implementation

this version of QM, and promising avenues for further development.

5.2 Protocol of reversed photon echo in an optically thick medium (CRIB protocol)

In Refs [78, 198–201], the CRIB protocol was proposed via the example of storing a single-photon wave packet in an optically thick medium of atoms with inhomogeneous broadening caused by the presence of the Doppler effect and the hyperfine interaction of REIs in the crystal [200]. An auxiliary level was used to control the optical atomic coherence, which simplifies the experimental implementation of the CRIB protocol and its subsequent modifications and has found application in long-term quantum information storage under the name of the spin-wave protocol. Considered below in connection with the experiments performed is a version of inhomogeneous broadening often implemented in crystals with REIs [195, 196].

Figures 3 and 4 show the characteristic temporal and spectral features of the CRIB protocol and the chosen method of its experimental implementation, which contains several stages. First, a narrow spectral line is produced by transferring (removing) nonresonant atoms to auxiliary levels, cooling the sample to the desired temperature (helium temperatures when setting up experiments with REIs, Fig. 4b), and preparing the atoms in the working quantum state $|\Psi_a\rangle = \prod_{j=1}^N |1_j\rangle$ (see Fig. 3, where 1_j is the lower state of the j th atom). Then, an external electric/magnetic field is applied, causing a change in the resonant frequencies of the working atoms due to the Stark–Zeeman effect. Control of the inhomogeneous broadening of the absorption line required for the CRIB protocol is initiated in this way (Fig. 4c). The external field can be applied perpendicularly or along the direction of propagation of signal pulses in the sample [195, 196, 206]. This completes the preparation of the atomic system.

When the signal radiation (signal) is injected into the medium at the point in time $t = 0$, the general state of the signal and atoms is of the form $|\Psi\rangle_{\text{in}} = |\psi_f\rangle \otimes |\psi_a\rangle$, where $|\psi_f\rangle$ describes the state of the signal propagating along the wave vector $\mathbf{k}_s = \mathbf{k}_0$, whose carrier frequency coincides with the center of the atomic transition line. The signal duration δt_s is assumed

to be much shorter than the atomic phase relaxation time

$$\delta t_s \ll \frac{1}{\gamma} = T_2,$$

and its spectral width is smaller than the inhomogeneous broadening of the optical transition $\delta\omega_s \sim \delta t_s^{-1} \ll \Delta_{\text{in}}$. These conditions distinguish the CRIB protocol from the slow-light QM [70, 72, 207], where the lifetime of optical quantum coherence can be shorter than the duration of the signal pulse. Below, we consider the QM for weak signal pulses: when they are absorbed by atoms, the population difference of the atomic states at the resonance transition $|1\rangle \rightarrow |3\rangle$ is assumed to change negligibly, $\langle\sigma_3^j(t)\rangle \cong -1/2$ [70, 122]. In this case, the MB-AB equations are linearized and coincide with the equations obtained for the wave function of atoms and the field in the case of a single-photon initial state of the signal [78].

Ignoring relaxation and Langevin forces during the signal action, system of Eqns (1) reduces to equations for the slow variables of the signal field $\hat{a}_s(Z, \tau)$ and the coherence of atoms $\sigma_-^j(\tau)$ (see Eqns (1)), where their full expressions have the forms $\hat{a}_s(Z, \tau) \exp(-i(\omega_0 t - k_s z))$ and $\sigma_-^j(\tau) \exp(-i(\omega_0 t - k_s z_j))$. The equations are conveniently solved using the variables of the moving coordinate system $Z = z, \tau = t - z/v_g$:

$$v_g \frac{\partial}{\partial Z} \hat{a}_s(Z, \tau) = i \frac{1}{2} \sum_{j=1}^N \Omega^*(\mathbf{r}_\perp^j) \sigma_-^j(Z_j, \tau) \delta(Z - z_j),$$

$$\frac{\partial}{\partial \tau} \sigma_-^j(\tau) = -i \Delta_j \sigma_-^j(\tau) + i \frac{\Omega(\mathbf{r}_\perp^j)}{2} \hat{a}_s(Z_j, \tau),$$
(7)

where the photon number density at the input to the medium $\langle \hat{n}_s(0, \tau) \rangle_{\text{in}} = \langle \psi_f | \hat{a}_s^\dagger(0, \tau) \hat{a}_s(0, \tau) | \psi_f \rangle$.

The best conditions for transferring the quantum state of a signal into atoms arise when the width of the resonance line described by the distribution function $G(\Delta/\Delta_{\text{in}})d\Delta$ significantly exceeds the spectral width of the signal pulse ($\Delta_{\text{in}} \gg \delta\omega_s$). We solve linear Eqns (7) to find expressions for the field operators and the atomic coherence of the field after signal absorption ($\tau \gg \delta t_s$):

$$\hat{a}_s(Z, \tau) = \hat{a}_s^0(0, \tau) \exp\left(-\frac{1}{2} \alpha Z\right) + i \frac{1}{2} \int_0^Z dz' \exp\left(-\frac{1}{2} \alpha(Z - z')\right) \hat{F}_a(z', \tau),$$
(8)

$$\sigma_-^j(\tau) = \sigma_-^j(\tau_0) \exp(-i \Delta_j(\tau - \tau_0)) + i \sqrt{\frac{\pi}{2}} \Omega(\mathbf{r}_\perp^j) \tilde{a}_s^0(Z_j, \Delta_j) \exp(-i \Delta_j \tau),$$
(9)

$$\hat{F}_a(Z, \tau) = \sum_{j=1}^N \Omega^*(\mathbf{r}_\perp^j) \sigma_-^j(\tau_0) \exp(-i \Delta_j(\tau - \tau_0)) \delta(Z - z_j),$$
(10)

where $\alpha = (\pi S \rho \langle \Omega \rangle^2 / 2 v_g) G(0)$ is the resonance absorption coefficient, $\tilde{a}_s^0(z, \Delta) = (1/\sqrt{2\pi}) \int d\tau \hat{a}_s^0(z, \tau) \exp(i \Delta \tau)$ is the Fourier component of the signal field with coordinate z in the depth of the medium, and in expression (9), the term

$$-\exp(-i \Delta_j \tau) \sum_{j' (z_{j'} = z_j)} \frac{\Omega(\mathbf{r}_\perp^j) \Omega^*(\mathbf{r}_\perp^{j'})}{8} \int_{\tau_0}^{\tau} d\tau' \sigma_-^{j'}(\tau') \exp(i(\Delta_j - \Delta_{j'}) \tau').$$

is omitted.

Assuming $z_{j'} \neq z_j$, we single out the term describing the self-action of the j th atom, which determines the slower irreversible decay of atomic coherence compared to the collective response of the atom. Disregarding its influence at the stage of interaction with the signal, we take into account the stronger collective response of the atoms. In deriving the solution, we made the transition from summation over atoms to integration over the inhomogeneous broadening of atoms located near the point in space Z [59, 205].

The first term in expression (8) describes the absorption of the input signal for a sufficiently high optical thickness ($\alpha L \gg 1$), so that its spectral components $\tilde{a}_s(Z_j, A_j)$ are transferred to the spectral components of the atomic coherence (9) of atoms. Upon completion of the signal absorption, it goes into free evolution $\sigma_-^j(\tau > \delta t_s) \sim \tilde{a}_s(Z_j, A_j) \exp(-iA_j\tau)$. At the next stage of the protocol, we transfer the optical coherence $\sigma_-^j(\tau)$ to the long-lived spin coherence $\sigma_{12}^j(\tau)$ of the atomic transition $|1_j\rangle \leftrightarrow |2_j\rangle$, acting with an additional (first) laser π pulse at the point in time $t_1 > \delta t_s$. This pulse must have a wave vector parallel to the signal radiation, $\mathbf{k}_1 \uparrow \mathbf{k}_s$, at resonance with the atomic transition $|3\rangle \leftrightarrow |2\rangle$, so that it is hardly absorbed in the medium.

An efficient transfer of optical coherence to spin coherence within the entire spectrum of excited atoms can be achieved using a high-power very short ($\delta t_1 \ll \delta t_s$) or chirped laser π pulse, especially efficient for transferring optical coherence for a larger spread of frequency detunings A_j [208, 209], due to which such pulses have found application in multipulse nuclear magnetic resonance [210, 211], quantum processing [212], and optical QM [213, 214]. The transfer of optical coherence $\sigma_-^j(\tau)$ to spin coherence ($\sigma_{12}^j = i \exp(-i\varphi_1^j) \sigma_-^j$, where $\varphi_1^j = \varphi_1 + \mathbf{k}_1 \mathbf{r}_j$, φ_1 is the phase of the laser pulse) occurs in a given intense classical field without forming macroscopic coherence and, accordingly, without coherent emission of photons by the atoms. This completes the stage of recording the signal pulse into long-lived spin levels of the atoms.

The stage of long-term storage of the quantum state of light in matter is provided by strong suppression of the phase relaxation of spin coherence. This problem is solved by using the so-called atomic clock transitions [48, 215], which are insensitive to weak local fluctuations of magnetic fields, as well as by using multi-pulse methods of dynamic averaging of the effect of such fluctuations [151, 216–219]. This technique continues to be improved with the inclusion of specificity of dipole-dipole interactions [220] and their influence in the presence of inhomogeneous broadening of the resonance transition [221–224].

After storage time T , the spin coherence acquires an additional phase $\sigma_{12}^j(\tau_2) = i \exp\{-i[\varphi_1^j \delta_0 T + \delta\phi_j(T)]\} \sigma_-^j(\tau_1)$ (δ_0 is the spin transition frequency shift, $\delta\phi_j(T)$ is the random phase incursion in the j th atom). Next (at the point in time τ_2), a short ($\delta t_2 \ll \delta t_s$) second laser pulse, which propagates in the direction opposite to the signal pulse (with wave vector $\mathbf{k}_2 \uparrow \mathbf{k}_s$), is applied to the medium to read out the recorded information. Before applying this pulse, we change the frequency detunings of each atom at the optical transition $A_j \rightarrow -A_j$ to the opposite ones (Fig. 4d, see above). For simplicity of consideration, we also assume that the transition frequency between the lower levels $\omega_{21}L/c \ll 1$ is low enough. This condition ensures the fulfillment of wave synchronism when emitting an echo signal in the direction opposite to the propagation of the

signal pulse [199, 202]. The second laser pulse transforms the existing spin and optical coherence $\sigma_{12}^j(\tilde{\tau}_2)$, $\sigma_-^j(\tilde{\tau}_2)$ into their new values, which, after the action of the pulse ($\tilde{\tau}_2 + \delta t_2$), assume the form ($\tilde{\tau}_2 = \tau_2 + z/v_g$):

$$\sigma_-^j(\tilde{\tau}_2 + \delta t_2) = \sigma_-^j(\tilde{\tau}_2) \cos \frac{\theta_2}{2} + i \exp(i\varphi_2^j) \sigma_{12}^j(\tilde{\tau}_2) \sin \frac{\theta_2}{2}, \quad (11)$$

$$\sigma_{12}^j(\tilde{\tau}_2 + \delta t_2) = \sigma_{12}^j(\tilde{\tau}_2) \cos \frac{\theta_2}{2} + i \exp(-i\varphi_2^j) \sigma_-^j(\tilde{\tau}_2) \sin \frac{\theta_2}{2}, \quad (12)$$

where θ_2 is the pulse area at the atomic transition $|2\rangle \leftrightarrow |3\rangle$, $\varphi_2^j = \varphi_2 + \mathbf{k}_2 \mathbf{r}_j$, and φ_2 is the phase of the second control pulse at the location of the j th atom. We write out the second term in $\sigma_-^j(\tilde{\tau}_2 + \delta t_2)$ in more detail:

$$i \exp(i\varphi_2^j) \sigma_{12}^j(\tilde{\tau}_2) \sin \frac{\theta_2}{2} = -\exp(-i\delta\Phi_j(T)) \sin \frac{\theta_2}{2} \times \exp(-i\Phi_{12}) \exp(i(\mathbf{k}_2 - \mathbf{k}_1) \mathbf{r}_j) \sigma_-^j(\tau_1), \quad (13)$$

where $\Phi_{12} = \varphi_{1,2} + \delta_0 T$ is a constant phase shift ($\varphi_{1,2} = \varphi_1 - \varphi_2$) introduced by the interaction of atoms with the control pulses and their evolution during the storage of quantum information in the QM. Taking into account $\mathbf{k}_1 - \mathbf{k}_2 = \mathbf{k}_s$, we obtain an additional factor in expression (13), $\exp(i(\mathbf{k}_2 - \mathbf{k}_1) \mathbf{r}_j) = \exp(-2i\mathbf{k}_s \mathbf{r}_j)$, which, taken into account in the complete operators of atomic coherence and the field, we obtain for them the expressions $\hat{a}_s(z, t) \exp(-i(\omega_0 t + k_s z))$ and $\sigma_-^j(t) \exp(-i(\omega_0 t + k_s z))$; the factor $\exp(-i\delta\phi_j(T))$ describes the influence of phase fluctuations.

Let us move on to new variables $Z = z$, $\tilde{\tau} = t + z/v_g$ and ignore the short time of photon flight through the medium to obtain equations for the slowly varying operators of the echo signal field $\hat{a}_e(Z, \tilde{\tau})$ and the atomic coherence for $\tilde{\tau} > \tilde{\tau}_R = \tilde{\tau}_2 + \delta t_2$:

$$-v_g \frac{\partial}{\partial Z} \hat{a}_e(Z, \tilde{\tau}) = i \frac{1}{2} \sum_{j=1}^N \Omega^*(\mathbf{r}_j^i) \sigma_-^j(\tilde{\tau}) \delta(Z - z_j), \quad (14)$$

$$\frac{\partial}{\partial \tilde{\tau}} \sigma_-^j(\tilde{\tau}) = i A_j \sigma_-^j(\tilde{\tau}) + i \frac{\Omega(\mathbf{r}_j^i)}{2} \hat{a}_e(Z_j, \tilde{\tau}), \quad (15)$$

where $\sigma_-^j(\tilde{\tau}_R = \tilde{\tau}_2 + \delta t_2)$ is given in expression (13), and the light field in the echo signal mode emitted in the opposite direction to the signal pulse (e -mode) is initially in the vacuum state $|0\rangle_s$ (where $n_e(Z, \tilde{\tau}) = \langle \hat{a}_e^+(Z, \tilde{\tau}) \hat{a}_e(Z, \tilde{\tau}) \rangle = 0$ for $\tilde{\tau} < \tilde{\tau}_2$), which determines the general initial quantum field and $|\Psi_v\rangle_{\text{in}} = |0\rangle_e \otimes |\psi_f\rangle \otimes |\psi_a\rangle$ in the Heisenberg representation. We solve Eqns (15) and (14) to obtain the emitted echo signal

$$\begin{aligned} \hat{a}_e(Z, \tilde{\tau}) = & -\Gamma_R(T) \sin \frac{\theta_2}{2} \exp(-i\Phi_{12}) \hat{a}_s^0(0, t_e - \tilde{\tau}) \\ & \times \exp\left(\frac{1}{2} \alpha Z\right) \{\exp(-\alpha Z) - \exp(-\alpha L)\} \\ & + \exp\left(\frac{1}{2} \alpha(Z - L)\right) \hat{a}_{0e}(L, \tilde{\tau}) + \hat{a}_{e,a}(Z, \tilde{\tau}) \end{aligned} \quad (16)$$

and the coherence of atoms after emission of the echo signal

$$\begin{aligned} \sigma_-^j(\tau) = & -\sin \frac{\theta_2}{2} [\exp(-i\delta\phi_j(T)) - \Gamma_R(T) \\ & \times (1 - \exp(-\alpha(L-z)))] \exp(-i\Phi_{12}) \\ & \times \exp(iA_j(\tau - \tau_R)) \sigma_-^j(\tau_1) + \sigma_-^j(\tilde{\tau}_2) \cos \frac{\theta_2}{2} \\ & \times \exp(iA_j(\tau - \tau_R)), \end{aligned} \quad (17)$$

$$\begin{aligned} \sigma_{12}^j(\tau) = & i \cos \frac{\theta_2}{2} \exp\{-i\delta_0(\tau - \tau_R + T) \\ & + i[\phi_1^j + \delta\phi_j(\tau - \tau_R + T)]\} \sigma_-^j(\tau_1), \end{aligned} \quad (18)$$

where $\Gamma_R(T) = \langle \exp(-i\delta\phi_j(T)) \rangle$ is a relaxation term, which often has the form $\Gamma_R(T) \cong \exp(-\gamma_m T)$, $t_e = \tilde{\tau}_R + t_1$ is the time of echo signal appearance, and $\sigma_-^j(\tau_1) = i\sqrt{\pi/2} \Omega(\mathbf{r}_\perp^j) \hat{a}_s^0(0, A_j) \exp(-iA_j\tau_1) \exp(-(1/2)\alpha Z_j)$, the second term $\exp((1/2)\alpha(Z-L))\hat{a}_{0e}(L, \tilde{\tau})$ describing the attenuation of the input field. In our case, in view of the input state $|\Psi_v\rangle_{\text{in}}$, it is in the vacuum state and therefore does not contribute to the signal at $z = 0$ for $\alpha L \gg 1$. The third term $\hat{a}_{e,a}(Z, \tilde{\tau})$ is associated with the part of the optical atomic coherence remaining after the action of the second control pulse; it corresponds to the state of atoms not associated with the action of the signal pulse, so it does not contribute to the echo signal $\langle \hat{a}_{e,a}(Z, \tilde{\tau}) \rangle = 0$, and therefore we will not take it into account further. The first term in expression (16) describes the echo signal emitted by the atoms for $Z < 0$:

$$\begin{aligned} \hat{a}_e(Z, \tau) = & -\Gamma_R(T) \sin \frac{\theta_2}{2} \exp(-i\Phi_{12}) \hat{a}_s^0(0, t_e - \tau) \\ & \times \{1 - \exp(-\alpha L)\}. \end{aligned} \quad (19)$$

In the absence of phase relaxation ($\Gamma_R^2(T) \cong 1$, $\delta\phi_j(T) = 0$) and for $\alpha L \gg 1$, the solution $\hat{a}_e(Z, \tau) = -\sin(\theta_2/2) \exp(-i\Phi_{12}) \hat{a}_s^0(0, t_e - \tau)$ reflects a time-reversible scenario of echo signal emission. For $\theta_2 = \pi$, the signal field is completely restored when all the atomic coherence transferred by the signal to the optical transition $\sigma_{13}^j(\tau)$ completely disappears when generating a photon echo signal. The photon echo signal acquires a phase Φ_{12} , which comprises the phase difference of the control laser pulses $\phi_{1,2}$ and the phase accrued at the spin transition of the atoms during the storage time $\delta_0 T$.

An important condition for efficient echo signal emission is the matching of the coherence amplitude with the magnitude of the absorption coefficient of the medium. As shown in Ref. [206], the imbalance between the magnitude of the absorption coefficient and the amplitude of the phasing polarization, which was controlled by introducing a scale factor in the inversion of atomic frequencies $\Delta \rightarrow -\eta\Delta$ ($\eta > 0, \eta \neq 1$), results in a decrease in the emission probability of an echo signal for the considered (transverse in nature) inhomogeneous broadening ($\sim 4\eta/(\eta+1)$) but does not affect the efficiency when the inhomogeneous broadening has a longitudinal nature, when the absorption of the echo signal is strongly suppressed.

In the above analysis of the properties, we restricted ourselves to consideration of the case when the spectral width of the signal pulse is much smaller than the inhomoge-

neous broadening of the resonance transition. However, the efficiency of the CRIB protocol does not lower if these quantities become comparable, $\delta\omega_s \approx \Delta_{\text{in}}$. An important requirement is complete absorption of the signal pulse in the medium, for which the medium must have a sufficiently high optical thickness at all spectral components of the signal pulse $\alpha(\Delta)L \gg 1$ ($\alpha(\Delta)$ is the resonance absorption coefficient at the frequency offset from the line center and the signal carrier frequency). As should be noted here, when a broadband signal pulse propagates in a medium (when $\delta\omega_s \approx \Delta_{\text{in}}$), such a pulse experiences the strong influence of spectral dispersion, which entails distortion of its temporal shape. However, as shown in Ref. [199], in the CRIB protocol, such dispersion effects are completely cancelled out at the echo signal output from the medium due to the temporal reversibility of this protocol.

The finite density of the medium ($\alpha L \ll 1$) and atomic decoherence $\Gamma_R^2(T)$ reduce the probability of echo signal emission. Deviation of the pulse area of the readout pulse θ_2 from π reduces the probability of echo signal emission and leaves some quantum information in the medium. This gives rise to quantum entanglement between the emitted echo signal and atomic coherence. As shown for a single-photon signal pulse, the remaining spin coherence can be restored in another echo signal by irradiating the medium with an additional readout laser pulse with pulse area π . The field of the second echo signal emitted at the point in time t_{e2} assumes the form for $\alpha L \gg 1$: $\hat{a}_{e2}(Z, \tau) = -\Gamma_R(T + \tau_{32}) \times \cos(\theta_2/2) \exp(-i\Phi_{13}) \hat{a}_s^0(0, t_{e2} - \tau)$, where τ_{32} is an additional delay time and $\Phi_{13} = \phi_{1,3} + \delta_0(T + \tau_{32})$. Two emitted echo signals completely restore the quantum information recorded in the medium with weak atomic relaxation. The echo signals will be in an entangled quantum state if the original signal radiation was in a quantum state. Studying and using such signal radiation readout modes is of interest for the efficient generation of new quantum states of light.

The CRIB protocol was implemented in an experiment where the frequency detuning inversion $\Delta_j \rightarrow -\Delta_j$ was provided by a switchable gradient of the electric field along the medium, causing a linear Stark effect in an REI-doped crystal and thereby producing a 'longitudinal' character of the inhomogeneous broadening of the optical transition. At the stage of recording the signal pulse, the frequency detunings are specified as $\Delta = \chi z$, where $-L/2 < z < L/2$. Furthermore, to preserve the signal pulse, the inhomogeneous broadening of the resonant transition must exceed the spectral width of the pulse: $\chi L \gg \delta\omega_s$. Before restoring the echo signal, the frequency detuning is inverted, $\Delta = -\chi z$, which was first demonstrated in Ref. [196]. Numerical experiments in Ref. [225] shown that the efficiency for this version of frequency detuning control may range up to almost 100% not only in the reverse direction, signal restoration, but also in the forward direction; this was named the CRIB/GEM protocol. The indicated possibility arose due to the exclusion of the effect of reabsorption by atoms during the propagation of the echo signal in the medium due to the fact that the atoms resonant to the field in this case are in a narrow space region, in the limit at one point of the medium. However, as was then shown in Ref. [226], due to the departure from time reversibility in this protocol, the echo signal, when radiated in the forward direction, can acquire a frequency chirp, which, however, can be largely suppressed if the signal pulse-medium interaction time becomes much longer than the pulse duration.

As shown in Refs [226] and [227], the behavior of the GEM protocol efficiency is similar to the efficiency of the CRIB protocol (6)–(19), where $\kappa = (2\pi^2/\chi v_g)n_0 S|g_0|^2$ acts as the optical thickness instead of αL (where n_0 is the atomic density, S is the cross-sectional area, and $g_0 = \sqrt{\omega_0/2\varepsilon_0\hbar}S$ is the photon-atom interaction constant). The GEM protocol has been repeatedly demonstrated experimentally in solids while storing quantum states of light [197, 228]. In Ref. [228], an efficiency of 69% was achieved for a $\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$ ($c = 0.005\%$) crystal in storing quantum states of light with a low noise level. The implementation of the GEM protocol by using the Zeeman effect in a switchable magnetic field gradient was demonstrated in three-level atomic gases at a off-resonant Raman transition [229–232]. In these studies, rubidium vapor gas was employed and an efficiency of 87% was achieved, which remains the best in the implementation of optical QM. A notable consequence of the highly efficient emission of the echo signal in the GEM protocol is the possibility of recording a signal field with a complex wave front [233–235], since the problem of wave synchronism does not arise in the emission of an echo signal in the forward direction. The CRIB/GEM protocol, along with the EIT protocol, is therefore of interest for the development of optical quantum holography, which is reliant on it. A disadvantage of the GEM protocol is the difficulty of achieving high optical thickness in a wide spectral range $\kappa = (2\pi^2/\chi v_g)n_0 S|g_0|^2$, since an increase in the working spectral range χL due to an increase in the gradient χ reduces the optical thickness of the medium κ .

The above discussion of the CRIB and GEM protocols was carried out for weak light fields. At the same time, it seems increasingly important to implement the QM for high-intensity light fields. For example, multiphoton bright light fields are employed in optical quantum communications [236], optical quantum computers, and nonlinear quantum metrology [237]. Below, we briefly dwell on the features of the QM for high-intensity light fields for the photon echo. This issue was first studied in Ref. [201] for storing intense classical signal fields in the CRIB protocol using the area theorem for the photon echo. We restrict the analysis of Ref. [201] to the assumption that two control laser pulses have a pulse area π necessary for efficient storage of the signal pulse in long-lived spin states and its subsequent restoration.

Considering that in the CRIB protocol the echo signal is emitted in the backward direction, then, according to Eqn (4), the equation for the pulse area $\theta_{\text{crib}}(z)$ will have the form

$$-\frac{\partial}{\partial z} \theta_{\text{crib}}(z) = \frac{\alpha}{2} \left(2P_e(z, 0) \cos^2 \frac{\theta_{\text{crib}}(z)}{2} + W_e(z, 0) \sin \theta_{\text{crib}}(z) \right), \quad (20)$$

where the radiation source $P_e(z, 0) = \Gamma_R(T) \sin \theta_s(z)$ and the inversion $W_e(z, 0) = -\cos \theta_s(z)$.

We solve Eqn (20) to find $\theta_{\text{crib}}(z)$, which has the following form at the medium output:

$$\theta_{\text{crib}}(z=0) = 2 \arctan \left(\Gamma_R(T) \tan \frac{\theta_s(0)}{2} \times \frac{1 - \exp(-\alpha L)}{1 + \tan^2(\theta_s(0)/2) \exp(-\alpha L)} \right). \quad (21)$$

Solution (21) generalizes solution (19) for signal fields of small pulse area ($\theta_s(0) \ll 1$) and describes the possibility of perfect reconstruction of the signal pulse area

$\theta_{\text{crib}}(z=0) = \theta_s(0)$. This takes place for weak phase relaxation $\Gamma_R(T) \cong 1$ and when more general requirements for the optical thickness of the medium are met: $\alpha L \gg 1$ and $\alpha L \gg \ln[\tan^2(\theta_s(0)/2)]$, which for $\theta_s(0) \ll 1$ are reduced to one known condition, $\alpha L \gg 1$. That is, the efficient storage of intense fields with a pulse area $\theta_s(0)$ close to π calls for the use of a higher optical thickness of the medium.

So, the area theorem shows the possibility of and conditions for using the CRIB protocol to efficient store high-intensity classical fields. It should be expected that solution (21) will also reflect the main properties in storing intense quantum fields. However, to study the quantum properties and memorize such fields, for example, optical quantum solitons [238], it is necessary to turn to the solution of quantum MB–AB equations (1), whose analytical solutions so far are limited to a search for the properties of optical solitons generated at a depth of the medium [99]. Nevertheless, an analysis of a number of properties of the CRIB protocol can be carried out when the intense input signal pulses are completely absorbed in the medium, which is also possible with a quantum description of radiation [99, 100]. In this case, after the end of the interaction, the light field passes into a pseudovacuum state (Fock vacuum $|0\rangle$) and the atoms into an excited nonradiating state. It is also noteworthy that, in a more detailed description, for example, when considering mesoscopic atomic ensembles with a low atomic density, the light radiation does not completely pass into a pseudovacuum state, and the photon scattering by atoms continues in the atomic medium [107]. Such a state is a version of a slow polariton containing an atomic and photonic component, similar to what occurs in the description of slow light [70]. The presence of a photonic component of a polariton will reduce the efficiency of the QM, but in an optically thick medium its effect will be insignificant [118].

Restricting ourselves to the macroscopic quantum MB–AB equations, we do not take into account the slower relaxation processes. A general analysis of the properties of the QM can be performed using the symmetry of these CRIB protocol equations by replacing the field operator of the signal pulse $\hat{a}(z, t)$ with the field operator of the signal echo with the signal $-\hat{a}_{\text{crib}}(z, t)$ (see discussion (19)) and inverting the frequency detunings $\Delta_j \rightarrow -\Delta_j$ and $z \rightarrow -z$. After such transformations, we obtain the equations for the stage of echo signal emission:

$$\begin{aligned} \left(-\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) \hat{a}_{\text{crib}}(z, t) &= i \frac{1}{2} \sum_{j=1}^N \Omega^*(\mathbf{r}_\perp^j) \sigma_-^j(z_j, t) \delta(z - z_j), \\ \frac{\partial}{\partial t} \sigma_-^j(t) &= i \Delta_j \sigma_-^j(t) + i \frac{\Omega(\mathbf{r}_\perp^j)}{2} \hat{a}_{\text{crib}}(z_j, t) \sigma_3^j(t), \end{aligned} \quad (22)$$

$$\frac{\partial}{\partial t} \sigma_+^j(t) = -i \Delta_j \sigma_+^j(t) - i \frac{\Omega^*(\mathbf{r}_\perp^j)}{2} \hat{a}_{\text{crib}}^+(z_j, t) \sigma_3^j(t),$$

$$\frac{\partial}{\partial t} \sigma_3^j(t) = i \frac{\Omega}{2} [\hat{a}_{\text{crib}}^+(z_j, t) \sigma_-^j(t) - \sigma_+^j(t) \hat{a}_{\text{crib}}(z_j, t)].$$

Comparing Eqns (22) with Eqns (1) of the signal pulse absorption stage, we see that they coincide if we change the time sign to the opposite one ($t \rightarrow -t$) in Eqns (22). This means that, if the initial state is prepared as atomic coherence arising after the absorption of the signal pulse, but with a reverse wave vector and phasing in time, Eqns (22) will describe the dynamics of atoms and light, the opposite of that which took place during the absorption of the signal

pulse. In this case, an echo signal in the form of a time-reversed copy of the signal field will be emitted at the output of the medium ($Z < 0$). This result generalizes the possibilities of implementing the reversible CRIB protocol, which was obtained earlier in Refs [173, 195, 204], to its formation in a single-mode optical waveguide. The use of the symmetry of the CRIB protocol equations also makes it possible to find the wave function of the emitted photon echo signals for an arbitrarily given initial quantum state of the signal pulse, as shown in Ref. [203]. However, the properties and possibilities of efficient multiple readout of high-intensity signal radiation remain unclear.

So, the implementation of the CRIB protocol with storage of the high-intensity signal pulse is based only on its absorption by atoms and does not depend on the relative width of the signal pulse or the magnitude of the inhomogeneous broadening of the resonance line: under conditions of strong nonlinear interaction, the effects of spectral dispersion will still be completely compensated in the emitted echo signal. It is worth noting here that the solution of the area theorem complements the direct use of the symmetry of the quantum MB-AB equations and clarifies the nature of the behavior of the echo signal in the medium under more general conditions (for low optical thickness, in the presence of phase relaxation), while allowing one to obtain an analytical solution for high-intensity plane waves, i.e., generalizing the analysis of the CRIB protocol properties. The symmetry of the CRIB protocol equations was also used to analyze its implementation using the off-resonant Raman approach in Ref. [204]. More recently, in [173], it was found that the use of a control laser field in such a version of the CRIB protocol implementation opens up the possibility of changing the carrier frequency and duration of the echo signal. These ideas are directly applicable to the implementation of the CRIB/GEM protocol with the emission of the echo signal in the backward direction, and they can work with high accuracy when emitting the echo in the forward direction. Many versions of the CRIB protocol have yet to be implemented experimentally.

It is pertinent to note that the approach developed in the CRIB protocol, which involves the use of the inverted dynamics of signal pulse recovery, was later successfully applied to the EIT [74], ATS [75, 125], ORS [172], and SR [167] protocols (see above). The CRIB and GEM protocols demonstrated the possibility of implementing broadband QM reliant on the inhomogeneous broadening of the atomic transition, a new degree of freedom for controlling the macroscopic atomic coherence. Somewhat later, other options for implementing inhomogeneous broadening and controlling the macroscopic atomic coherence were proposed, which broadened the possibilities for implementing optical QM. However, these protocols, a description of which we move to below, already depart from the exact observance of time reversibility, which facilitates their experimental implementation and can increase the information capacity, but causes problems in the accuracy of restoring the original quantum states, requiring correction and limitation of the physical conditions for the implementation of these protocols.

5.3 Protocol based on atomic frequency comb (AFC protocol)

In 1986, Dubetskii and Chebotaev [239, 240] proposed and experimentally demonstrated the echo effect in a system of nonlinear oscillators whose frequencies have the form of

periodically spaced narrow lines $\Delta_n = n\Delta$ with a period Δ . After excitation at the point in time $t = 0$, the emerging coherence of the atoms of the n th group begins to oscillate with its frequency $\sim \exp(-in\Delta t)$, so that, at the points in time $t_n = 2\pi n/\Delta$, the phases of all atoms coincide again, resulting in the automatic appearance of signals. The authors called this effect imaginary echo and also demonstrated it experimentally. In 2008, in Ref. [241], this version of inhomogeneous line broadening was applied to the above photon echo QM scheme instead of frequency detuning inversion and received the name Atomic Frequency Comb (AFC) protocol. Compared to the CRIB/GEM protocol, the new protocol enables a significant increase in the working spectral width and, accordingly, an increase in the information capacity of the QM. Due to this, the AFC protocol holds the greatest interest today in the experimental development of a quantum repeater.

Significant differences with respect to CRIB/GEM protocols arise from the nature of inhomogeneous broadening, with the result that the AFC protocol loses full reversibility in signal radiation recovery, but also provides advantages in its implementation. There are also problems in achieving high efficiency and a number of new properties that deserve attention. Figure 5 shows typical diagrams of the formation of the AFC protocol in a two- and three-level scheme.

The experimentally implemented AFC structure of inhomogeneous broadening of the resonance transition is a periodic system of N narrow absorption peaks with a given frequency interval Δ and the width of each peak γ , which, as a rule, significantly exceeds the homogeneous broadening of the optical transition $\gamma > 1/T_2$. In practice, the AFC structure is produced using a well-developed technique of laser burning of spectral holes in the inhomogeneously broadened absorption line. In the experiments implemented, the spectral width Δ_{afc} is usually much narrower than the natural inhomogeneous broadening of the line ($\Delta_{\text{afc}} \ll \Delta_{\text{in}}$), although of major interest is the case of the greatest spectral width Δ_{afc} . A significant consequence of the AFC structure production is a decrease in the effective optical thickness of the resonance transition by a factor γ/Δ , although this decrease is $1/N$ that in the CRIB protocol in the implementation with the same working spectral range. Nevertheless, the appearance of the factor γ/Δ can lead to a strong decrease in the efficiency of the AFC protocol. However, these efficiency problems are solved by accommodating the crystal in an optical cavity with optimally selected parameters in the so-called impedance

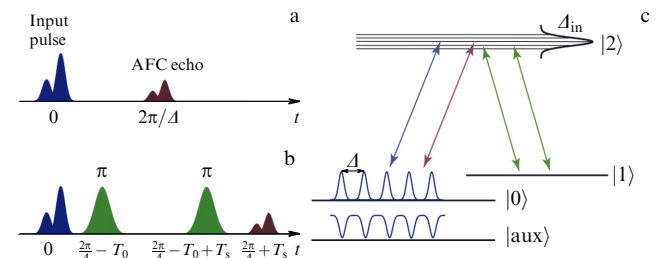


Figure 5. (a) Time sequence of AFC protocol in two-level medium with formation of an echo signal after time $t_e = 2\pi/\Delta$. (b) AFC protocol involving a 'spin wave' with two control laser pulses, which serve to increase signal storage time for lifetime of QM by storing signal in spin coherence of lower levels $|0\rangle$ and $|1\rangle$; $|aux\rangle$ level is auxiliary, serving to prepare AFC structure.

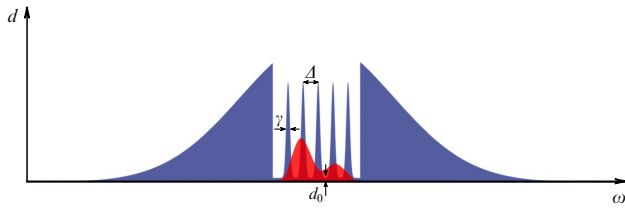


Figure 6. Typical AFC structure of resonance lines prepared in a natural inhomogeneously broadened optical transition line. d is spectral optical thickness of optical transition, Δ is spectral interval between nearest lines, γ is line width of an individual peak, d_0 is residual background absorption level. Spectrum of signal pulse is shown in red.

matching [242–246]. A description of the AFC protocol operation relies on the same system of Eqns (1), (3), (7), where the nature of inhomogeneous broadening is taken into account. We dwell on the most important properties of the AFC protocol.

As in any photon echo QM protocol, the signal pulse is launched into the medium with a spectral width much smaller than the spectrum of the produced frequency comb $\Delta_{\text{afc}} = N\Delta$, as shown in Fig. 6. Under conditions where the signal spectrum is much broader than the width of an individual peak, $\delta\omega_s \gg \gamma$, the presence of this structure does not manifest itself during the interaction of such a pulse with atoms possessing a fine AFC structure of narrow lines. This important (positive) effect is due to the fact that, at time intervals of the order of the pulse duration, the atoms manifest themselves via the averaged optical transition thickness $(\gamma/\Delta)\alpha L$. If it is high enough, $(\gamma/\Delta)\alpha L \gg 1$, the light signal is absorbed. The AFC structure of the spectrum manifests itself only later, when the oscillating excited atoms begin to phase, restoring the macroscopic coherence by the point in time $t_e = 2\pi/\Delta$, resulting in the generation of the echo signal. So, the echo signal will be automatically emitted in the same direction in which the signal pulse propagated. In this case, the temporal shape of the echo signal repeats the temporal shape of the signal pulse, without its reversal, which is the case with the CRIB protocol.

The solution of the MB–AB equations for a weak narrowband echo signal emitted in the same direction for $\delta\omega_s \ll \Delta_{\text{afc}}$ yields its radiation efficiency $(\alpha L)^2 \exp(-\alpha L)$, which does not exceed 54% for $\alpha L = 2$. This is due to the reabsorption of the echo signal in the medium, which also distinguishes this version of echo signal reconstruction. The presence of phase relaxation of the atomic transition reduces this efficiency similarly to the usual primary echo, but the greatest negative impact on efficiency is caused by the limited AFC structure sharpness Δ/γ [241]. It is possible to lengthen the QM lifetime by introducing additional controlled dephasing and phasing of optical atomic coherence, for instance, by placing atoms in a gradient external electric or magnetic field and observing the echo signal at the point in time $nt_e = 2\pi/\Delta$ ($n = 2, 3, \dots$) with the same efficiency [247, 248]. When it is necessary to significantly improve the efficiency and lifetime of the signal in the quantum field, it is possible to transfer optical quantum coherence to spin coherence, and then return it by using two controlling laser π -pulses, similar to the CRIB protocol [78]. Such experiments were recently implemented in Refs [244, 249]. In this case, the highest efficiency of the AFC protocol can reach a figure close to 100%. However, so high an efficiency ($> 90\%$), as shown in Ref. [250], is possible when the spectral width of the signal is

much smaller than the inhomogeneous broadening of the AFC structure, $\delta\omega_s \ll \Delta_{\text{afc}}$. If the signal light pulses have a spectral width comparable to the spectral width of the AFC structure, dispersion effects in the signal emission begin to negatively manifest themselves during its propagation in an optically thick medium, reducing the efficiency of the emission and the fidelity of the signal pulse reconstruction [250]. The occurrence of such an effect is due to a fundamental reason: the absence of strict temporal reversibility of the echo signal emission in relation to the signal pulse absorption.

As shown in Refs [251, 252], it is possible to achieve a high efficiency of the AFC protocol in some widest possible working spectral frequency range, which is only slightly narrower than the spectral width $\Delta_{\text{qm}} < 0.9\Delta_{\text{afc}}$. This is achieved by choosing the optimal spectral width of the AFC structure Δ_{afc} , which depends on the required protocol efficiency, the initial value of the inhomogeneous line broadening Δ_{in} , and the optical thickness of resonant transition αL . In this case, the AFC protocol efficiency of 90% in the working frequency range Δ_{qm} is achieved due to almost complete compensation of the spectral dispersion.

The AFC protocol was experimentally implemented for different crystals with REIs and different optical transition wavelengths [249, 253–257]. Using the AFC protocol, the storage of more than 1000 temporal modes [258], a high QM efficiency of 62% [245], and a storage time of 1 hour [219] have been demonstrated to date. The broadband AFC protocol ($N\Delta = 6$ GHz) was demonstrated [259]. The AFC protocol was implemented in an optical cavity [245], and an implementation of the AFC protocol in an integrated waveguide circuit was shown in a series of papers [56, 247, 260, 261]. To date, these results have demonstrated the great potential of this protocol for a low implementation efficiency, when the above-mentioned dispersion effects do not play a significant role. Intensive development of this protocol is underway, opening up hopes for its use in making a quantum repeater. AFC structures open up new possibilities for controlling not only the optical QM scheme described above: for example, in such structures it becomes possible to implement broadband slow light pulses [262]. Studying the properties of such light pulses can provide new possibilities for the control and quantum processing of single-photon fields. Of interest is the implementation of the AFC protocol in different physical systems: for example, for electron-nuclear spin transitions [263] or for a system of high-quality superconducting cavities [264], whose use can be of great importance for developing QM for a superconducting quantum computer.

5.4 Revival of silenced echo

The year 2011 saw the appearance of photon echo QM protocols oriented to the use of the natural inhomogeneous broadening of atomic transitions [213, 265, 266]. The most developed protocol at present was reported in Ref. [213], which is known as the ‘Revival of Silenced Echo’ (ROSE). Figure 7 shows a sequence of light pulses of the ROSE protocol (the level diagram is shown on the left side of the figure). In this protocol, an additional rephasing π -pulse is added to the sequence of two-pulse photon echo, with the consequence that the resultant photon (ROSE) echo signal is emitted in a noninverted system of atoms without the appearance of quantum noise and becomes suitable for restoring the quantum information of the first signal pulse. Therefore, the ROSE protocol relies on the principles of conventional photon echo and uses the ‘natural’ inhomoge-

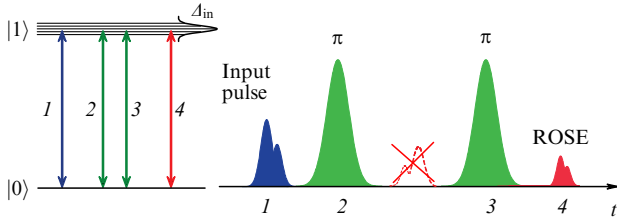


Figure 7. QM protocol in the scheme of revival of silenced echo (ROSE).

neous broadening of the medium without preparing the initial quantum state, making it the simplest for experimental implementation. The indicated properties of the protocol provide important merits in its implementation. This protocol also allows implementing broadband QM with the use of a significant part of the inhomogeneously broadened optical transition line.

The implementation of the ROSE protocol implies suppression of the primary photon echo. This can be done by selecting the direction of propagation of the first control pulse, for example, orthogonal or antiparallel to the direction of propagation of the signal (first) pulse, whereby the phase matching condition for the emission of the primary echo is violated. The primary photon echo can also be suppressed by introducing controlled dephasing of the atomic coherence by applying a switchable external electric (magnetic) field [265, 267].

For high efficiency and long storage time in the ROSE protocol, both high optical thickness and long coherence time T_2 of the optical transition are necessary. For this purpose, REI-doped crystals seem to be the most promising due to their properties of homogeneous and inhomogeneous broadening of the lines of optical transitions inside the 4f shell [23]. Table 2 presents the results of the experimental implementation of the ROSE protocol with such crystals. This protocol was experimentally implemented mainly in the geometry of antiparallel propagation of the signal pulse and two control laser pulses $\mathbf{k}_1 \uparrow \downarrow \mathbf{k}_2 \parallel \mathbf{k}_3$ [205, 213, 268–271]. In this case, the direction of the wave vector of the emitted echo signal coincides with the direction of propagation of the input pulse ($\mathbf{k}_e = \mathbf{k}_1$), leading to reabsorption of the echo signal during propagation inside the medium and limiting the highest protocol efficiency to 54%. It is noteworthy that this

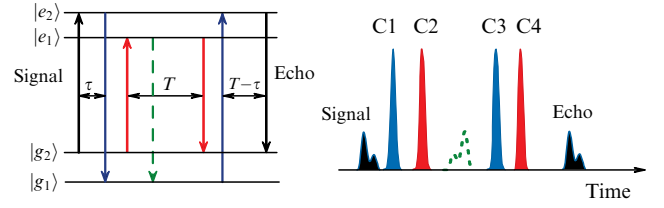


Figure 8. Temporal diagram of light pulses with appearance of echo signal and unwanted signal, which is indicated in green.

geometry was also used to implement the ROSE protocol in femtosecond laser-printed waveguide structures in $\text{Eu}^{3+}:\text{Y}_2\text{SiO}_5$ [270] and $\text{Tm}^{3+}:\text{YAG}$ [205] crystals.

An orthogonal geometry of the propagation of the signal and control fields was experimentally demonstrated in Refs [214, 272]: $\mathbf{k}_1 \perp \mathbf{k}_2 \parallel \mathbf{k}_3$, in which the suppressed echo signal is also emitted parallel to the signal pulse. This geometry allows a stronger suppression of optical noise caused by the action of strong laser pulses, and it is convenient for implementing the QM in an optical cavity, allowing a significant enhancement of the interaction of the signal field with matter and an increase in the QM efficiency up to $\eta = 100\%$. In Refs [273, 274] this protocol was implemented in an optical cavity in the impedance matching condition for an input signal recovery efficiency $\eta > 21\%$ and a storage time of 36 μs . The decrease in efficiency was due to the influence of phase relaxation of atoms and the difficulties of precise impedance matching. At the same time, address readout of signal pulses from a QM cell was demonstrated in Ref. [274]. The signal pulse contained an average of 5.6 photons for a signal-to-noise ratio of unity ($\text{SNR} = 1$).

It is pertinent to note that, despite all the advantages of the experimental implementation of the ROSE protocol, producing an area close to π for the rephasing pulses remains an intricate task. The imperfection of the rephasing pulses leads to the appearance of additional quantum noise in the echo signal, which is mainly caused by spontaneous emission of atoms. Significant attenuation of quantum noise in the ROSE protocol was recently demonstrated due to its implementation in a four-level scheme [276] (Fig. 8). In this case, an additional adjacent optical transition was used to restore atomic coherence at the working atomic transition g_2-e_2 .

Table 2. Characteristics of QM based on ROSE protocol on REI-doped crystals (c — $\mathbf{k}_e = \mathbf{k}_1$ w — implementation in a waveguide, r — implementation in a resonator, n — average number of photons in input pulse, SNR — signal-to-noise ratio).

Crystal	λ , nm	η QM	Storage time	Single-photon level	Geometry	References
$\text{Tm}^{3+}:\text{Y}_3\text{Al}_5\text{O}_{12}$	793	10%	26 μs	—	c	[213]
$\text{Tm}^{3+}:\text{Y}_3\text{Al}_5\text{O}_{12}$	793	12%	40 μs	$n = 14$ (SNR = 1)	c	[269]
$\text{Tm}^{3+}:\text{Y}_3\text{Al}_5\text{O}_{12}$	793	21%	36 μs	$n = 5.6$ (SNR = 1)	r	[273, 274]
$\text{Tm}^{3+}:\text{Y}_3\text{Al}_5\text{O}_{12}$	793	0.5%	30 μs	—	c + w	[205]
$\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$	606	20%	15 μs	—	c	[275]
$\text{Eu}^{3+}:\text{Y}_2\text{SiO}_5$	580	23%	8 μs	—	c + w	[270]
$\text{Er}^{3+}:\text{Y}_2\text{SiO}_5$	1536	12%	82 μs	—	c	[213]
$\text{Er}^{3+}:\text{Y}_2\text{SiO}_5$	1536	40%	16 μs	—	c	[271]
$\text{Er}^{3+}:\text{Y}_2\text{SiO}_5$	1536	44%	40 μs	$n = 340$ (SNR = 4)	c	[253]
$\text{Er}^{3+}:\text{Y}_2\text{SiO}_5$	1538	17%	60 μs	$n = 38$ (SNR = 1.3)	c	[272]

Spontaneous emission of atoms at this transition was blocked by a filtering crystal [276], due to which it was possible to increase the signal-to-noise ratio to 42.5 for a storage time of 22.5 μ s of a signal pulse containing $\bar{n} = 1.17$ photons on average, with an efficiency $\eta = 6.4\%$. It is noteworthy that, in the implemented QM scheme, in order to suppress quantum noise, it was necessary to empty one more level g_1 . This complicates the experimental implementation of the protocol, but the achieved results definitely demonstrate the practical significance of the ROSE protocol.

6. Application of optical quantum memory based on photon echo in quantum repeater protocol

As shown theoretically in Ref. [29], the use of a multimode optical QM can increase the rate of quantum entanglement generation by several orders of magnitude between repeater nodes and relax the requirements for the entanglement storage time. Instead of generating entanglement between atoms and light, as in the DLCZ protocol [153, 277], it was proposed to use an external source of photon pairs (biphotons) in an entangled quantum state. The requirement for the QM in this version of the repeater is the ability to record, store, and restore several independent modes at once, for example, temporal, spatial, or frequency modes to ensure the appropriate multiplexing. Another advantage of this type of repeater is the use of two-photon interference in Bell state measurement nodes, in contrast to the single-photon interference used in the DLCZ protocol. Single-photon interference imposes additional requirements for strict phase stability of the control lasers and the length of the fiber (interferometer), which is extremely difficult in practical applications for transmitting quantum information over long distances.

An external source of biphotons is often a nonlinear crystal in which the spontaneous parametric scattering (SPS) effect is realized. This effect is a nonlinear process of light interaction with an anisotropic crystal, in which a pair of photons is spontaneously generated from one photon of high-intensity pump radiation. In this case, quantum entanglement can be encoded into different degrees of freedom of biphotons, such as polarization, frequency, spatial or temporal modes, and orbital angular momentum, as well as into temporal bins (wave packets of single-photon states) [278]. The disadvantage is that biphotons have a very large spectral width (> 100 GHz), which significantly exceeds the bandwidth of most atomic transitions, including QM based on REI-doped crystals (1 MHz–10 GHz) [23]. One way to match the spectral width of biphotons with the working band of optical quantum memory is to use narrow-band spectral filtering in the form of an external optical element. However, in this case, the count rate of photon coincidences of generated photon pairs is significantly reduced.

Another approach to generating narrow-band photon pairs is to place a nonlinear crystal inside an optical cavity whose resonant frequencies coincide with the frequency of the generated biphotons [279]. In this configuration, the spectral width of the biphotons decreases, and the spectral brightness of the source increases. Such a configuration was implemented in Fabry-Perot optical cavities of various designs. However, to achieve a sufficiently narrow spectral linewidth (< 10 MHz) of the generated photon pairs, Fabry-Perot cavities are rather difficult to align. In this case, it is more convenient to employ ring-type volume cavities, with the help

of which biphoton sources with a spectral linewidth of up to 1 MHz were implemented [279].

We emphasize that, for the application of a photon pair source in a quantum repeater protocol, it is desirable that at least one of the photons in the pair have a wavelength in the telecommunication C-band in order to minimize losses in the optical fiber, while the second photon may have a different wavelength for storage in the QM. In Refs [280, 281], advantage was taken of a frequency-nondegenerate biphoton source, where the wavelength of the idler photon was $\lambda = 1436$ nm and the signal photon was stored in the QM cell with an operating wavelength $\lambda = 606$ nm.

Different media can be used for a QM cell. However, one of the most interesting platforms is REI-doped crystals, since their quantum transitions have a number of advantages in the experimental implementation of QM. REIs in such crystals exhibit an exceptionally narrow homogeneous line broadening and, accordingly, a long coherence time at cryogenic temperatures for transitions between levels within the 4f configuration due to screening by the outer 5s and 5p shells. In combination with a large inhomogeneous line broadening of the optical transition, this allows implementing broadband optical QM (> 1 GHz) in such crystals. Many REIs have isotopes with a nonzero nuclear spin, which can have a long quantum coherence lifetime. QM schemes can therefore be developed in the spin wave protocol using nuclear spin sublevels whose lifetimes can range up to days and coherence times to several hours [48, 282].

Currently, the main technique of optical QM used for quantum teleportation of the quantum state of a photon and a qubit in such crystals is the AFC protocol [241], discussed in detail in Section 5.3. The achieved values of the basic parameters of the AFC protocol have yet to be implemented together in a single QM device; solving this problem is important for making a practically significant optical QM. Nevertheless, the low level of quantum noise of the AFC protocol and the possibility of its implementation on a chip in a crystal stimulate the setting up of different experiments for its further improvement. Below, we discuss research in which quantum teleportation of the photon quantum state was experimentally implemented using the AFC protocol.

Quantum entanglement between two solid-state QM cells was experimentally demonstrated in Ref. [283]. Advantage was taken of entangled photon pairs in temporal modes, with photon wavelengths of 794 nm and 1535 nm in each pair. After separation of the photon propagation direction by wavelength, they were stored in a $\text{Tm}^{3+}:\text{LiNbO}_3$ crystal (transition wavelength: $\lambda = 794$ nm) and in an optical fiber doped with erbium ions Er^{3+} ($\lambda = 1535$ nm). Quantum entanglement between two different QM cells was obtained with an accuracy of $\sim 93\%$. Due to the low efficiency of such QM cells, which was 0.1% in the erbium-doped fiber and 0.4% for the $\text{Tm}^{3+}:\text{LiNbO}_3$ crystal, the probability of entanglement of the two QM cells was about 4×10^{-6} , which is extremely low for practical applications and lends impetus to further improvement of this QM.

The distribution of quantum entanglement between two QM cells with result notification was experimentally realized in Ref. [284]. The experimental setup consisted of two separate sources of polarization-entangled photon pairs, which interfered on a beam splitter with subsequent projection onto Bell states, which led to the production of polarization entanglement of two states that were stored in

separate QM cells located at a distance of 3.5 m. An $\text{Nd}^{3+}:\text{YVO}_4$ crystal (wavelength of the optical transition of neodymium ions $\lambda = 880$ nm) was chosen as the QM cell, in which an AFC structure with a storage time of 56 ns, a bandwidth of 1 GHz, a signal photon recovery efficiency $\eta_{\text{QM}} \approx 13\%$, and a quantum state fidelity $F_{\text{QM}} \approx 96\%$ was implemented. Bell states were detected at a frequency of 100 Hz and predicted entanglement of the two QM cells with each other. The resultant entanglement distribution frequency, taking into account all losses, was 1.1 per hour with a quantum state fidelity $F = 80.4\%$. This experiment is the closest to a scalable quantum repeater module.

Quantum teleportation over a distance of 1 km was experimentally realized between a photonic qubit at a telecommunication wavelength ($\lambda = 1.5$ μm) and a photonic qubit with a wavelength of 606 nm, which was stored in a $\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$ crystal [281]. In the case of successful interference of two photons at the telecommunication wavelength with subsequent measurement of the Bell state, the probability of qubit teleportation before (after) the QM cell for a short distance (5 m) was 7.1×10^{-2} (1.2×10^{-2}), while it became insignificantly lower for a distance of 1 km, being 6.5×10^{-2} (7.5×10^{-3}). The frequency of successful measurement of the Bell state was 1 Hz at $\eta_{\text{QM}} \approx 18\%$. In this experiment, a feedback system was additionally used to implement a conditional phase shift of the qubit extracted from the QM, as required by the quantum teleportation protocol. Time multiplexing was also demonstrated.

Note that the teleportation repetition rate is limited by the two-way communication time between two nodes. For instance, a distance of 1 km between nodes corresponds to a synchronization time of ~ 10 μs , equal to the QM storage time. Due to the temporal multimode nature of the QM, this limitation was overcome in this work, which made it possible to increase the teleportation frequency threefold. Unlike Ref. [284], this experiment used only one source of entangled photon pairs and demonstrated only teleportation between the quantum state of a photon at a telecommunication wavelength and a photon with a wavelength of 606 nm, which was stored in the QM cell. The use of such a scheme as a module of a quantum repeater invites the addition of a second source of entangled photon pairs.

7. Prospects

Described above were the most important characteristics of QM (fidelity and noiselessness, efficiency, lifetime, spectral width, multimode nature, operating wavelength, integrability). Papers concerned with the experimental implementation of QM, via the example of REI-doped crystals, in which the most significant results were obtained over the past few years, are collected in Table 3.

In our view, successful application of QM in a quantum repeater and obtaining a significant gain in the entanglement distribution speed due to the use of QM calls for simultaneous attainment of high values of all basic QM characteristics. The previously listed studies are devoted to the implementation of laboratory QM, in which high QM performance characteristics were achieved for only 1–2 parameters. Below are several striking examples for QM based on REI-doped crystals. For QM reliant on the AFC protocol in $\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$ [245], high efficiency (62%) was obtained, but with a storage time of 2 μs (tens of seconds to minutes are required for use in a quantum repeater), a small spectral width (no greater than 2 MHz), and a wavelength of 606 nm, requiring conversion to the telecommunication range with a corresponding $\sim 50\%$ loss. An AFC protocol for QM in $\text{Eu}^{3+}:\text{Y}_2\text{SiO}_5$ (580 nm) with a storage time of 60 min [219] was recently demonstrated, though with an efficiency of only 0.005%, where a dynamic decoupling protocol was used, for which the absence of an effect on the accuracy (fidelity) of the QM has not yet been proven. An important example is the AFC protocol for QM based on Er^{3+} in Y_2SiO_5 , since only this ion allows the making of QM in the telecommunication wavelength range (~ 1.5 μm). The best results are collected in Table 3 [253]. The low characteristics obtained are due to the fact that the erbium ion is a Kramers ion and has an electron spin, which significantly complicates the AFC structure preparation and requires work in high magnetic fields (~ 3 –7 T). Approaches based on the EIT or ROSE protocols can demonstrate relatively high efficiency ($\sim 40\%$), but high noise values in the ROSE protocol do not yet allow us to talk about the use of such a QM in a quantum repeater, for example, Refs [152, 271, 274, 285]. An interesting approach is to modify the ROSE protocol by using an additional level and an additional pulse sequence, which made it possible to

Table 3. Comparative table of latest experimental achievements in implementation of QM in REI-doped crystals (o — QM in an optical transition, s — QM in long-lived spin states).

Crystal	λ , nm	η QM (o/s)	Storage time	QM lifetime	Single-photon level	Protocol	Spectral width	Retrieval on demand	References
$\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$	606	62% (o)		89 μs	+	AFC + cavity	< 2 MHz	–	[245]
$\text{Eu}^{3+}:\text{Y}_2\text{SiO}_5$	580	0.005% (c)	60 min	52.9 min	–	AFC + DD	1 MHz	+	[219]
		4.5% (o)	10 μs	–					
$\text{Er}^{3+}:\text{Y}_2\text{SiO}_5$	1538	22% (o)	0.7 μs	–	+	AFC	5 MHz	–	[253]
$\text{Nd}^{3+}:\text{YVO}_4$	880	12.5% (o)	0.05 μs	–	+	AFC	1 GHz	–	[284]
$\text{Tm}^{3+}:\text{LiNbO}_3/\text{Er}^{3+}:\text{SiO}_2$	794/1535	2% (o)/ 0.5% (o)	32 ns/6 ns	–	+	AFC	10 GHz/ 8 GHz	–	[283]
$\text{Pr}^{3+}:\text{Y}_2\text{SiO}_5$	606	5.6%	1.28 s	7.5 s	$n = 10$ (SNR = 1)	EIT + DD	100 kHz	+	[152]
		42%	2 μs		$n = 1$ (SNR = 3)	EIT			

reduce quantum noise to approximately 30 times compared to the standard ROSE protocol [219]. In doing so, such a modification of the protocol led to a decrease in efficiency to approximately 10% with a storage time of 21.7 μ s.

A practically important line of development of optical QM protocols is their implementation in cavity and waveguide schemes. Placing an atomic ensemble in an optical cavity allows one to significantly relax the requirements for the optical transition thickness while fulfilling the impedance matching condition (the optical thickness of the atomic transition and the coupling constant of the cavity mode with the waveguide) [242, 286]. High efficiencies in implementing the AFC and ROSE protocols were achieved in this way [243–246, 273, 274]. This approach also seems to be especially promising for implementing photon echo QM with nonresonant Raman interaction in REI-doped crystals with a small optical transition width [287], since, in this case, it would be possible to immediately transfer photonic qubits to long-lived spin states. The best results along this path so far have been obtained for QM using confocal Fabry–Perot cavities. The use of miniature ring and nanophotonic cavities is a topical line of development [54, 261, 288, 289], where the achieved QM efficiency does not exceed 10%, which invites further research. The use of high-Q cavities can significantly facilitate the implementation of optical QM and provide the widest spectral range of their operation due to the implementation of zero spectral dispersion in this range [290]. Great hopes are pinned on the implementation of optical QM in waveguides produced in REI-doped crystals [54, 56, 205, 259, 291–293]. QM in such a scheme is convenient for integration into external optical waveguides. In this new area of research, it is necessary to significantly improve the QM efficiency, including the development of nondestructive methods for monitoring the quantum coherence of atoms, and to solve technological problems of manufacturing high-quality waveguides with the desired parameters (see review Refs [55, 294]).

Another important problem with QM for use in a quantum repeater is that most researchers report the so-called internal efficiency, i.e., the ratio of the number of photons recovered from the QM cell, for example, directly at the output from the crystal or gas cell, to the number of photons directly in front of the crystal or gas cell on average per pulse. Important for the construction of quantum repeater nodes is the efficiency of the entire optical QM scheme, or End-to-End efficiency of the QM, usually several times or even several orders of magnitude lower than the ‘internal’ efficiency. This is due to the weak integrability of the QM into fiber-optic or optical integrated circuits, and the nonideality of the optical circuit and its elements.

A similar problem exists for the sources of entangled photon pairs (EPPs) required for a quantum repeater. Since the QM in most cases has a small spectral width (units to tens of MHz), intricate techniques of intracavity SPS-based generation of EPPs are used to obtain EPPs with the corresponding spectrum. Existing studies [279, 295, 296] and our experience in this area [297] suggest that the End-to-End spectral brightness of such sources is still low for their use in multi-node quantum repeaters. In Ref. [284], for example, the entanglement distribution rate, with the inclusion of all losses, was 1.1 per hour. Such a low figure is primarily due to the low probability of generating entangled photon pairs in the SPS. The use of a deterministic entanglement source, for example, involving cascade emission of quantum dots [298–300] together with multiplexing, should significantly improve

these values. However, in this case, it will be necessary to develop an optical quantum frequency converter [301], which will make it possible to control the wavelength of emitted photons for their further storage at quantum transitions of the atom in the ensemble.

Today, QM remains an expensive laboratory device that requires sophisticated laser systems, cryogenic and vacuum equipment, and a wealth of optoelectronic and optomechanical devices. In the next few decades, QM should follow a path similar to classical digital memory: from magnetic core memory, vacuum tube memory, to modern semiconductor memory on a chip. To obtain QM as a practically significant device, large-scale research is being pursued by many scientific groups. We list the most important, in our opinion, areas in the necessary research and scientific-technical work.

(1) *Protocols*. Theoretical and experimental development of new and improvement of existing QM protocols. For example, the spin-wave AFC protocol invites methods for increasing the efficiency of coherence transfer from optical transitions to electron-nuclear ones and vice versa. The ROSE protocol requires finding methods for suppressing quantum noise. Almost all protocols require research to significantly improve the efficiency and broaden the spectral band of QM operation. For example, the implementation of optical quantum memory, which uses macroscopic coherence to retrieval stored signal fields, is of interest [302].*

(2) *Materials science*. The quest for and study of new materials for QM with higher values of coherence times of the transitions in use. For REI-doped crystals, a promising realm is the use of crystals with an isotopic composition of the ions of the crystal matrix without a nuclear spin. For the implementation of a promising nonresonant Raman QM protocol, an important area is the production of materials with an ultra-small inhomogeneous broadening at the optical transition (less than 10 MHz) and large coherence times at optically addressed electron-nuclear states. For warm gases, as noted above, a breakthrough area may rely on the use of ‘noble’ gases. Such QM involves the cascade transfer of quantum coherence from the atoms of the quantum interface (photons–atoms) to the nuclear spin states of the noble gas atoms located in the same cell as the interface atoms. Effecting this transfer requires a lot of time, so this QM is of interest for the knowingly long-term storage of photonic qubits [138].

(3) *Integration*. An important direction is the miniaturization of QMs and their efficient integration into fiber-optic and optical integrated circuits [55]. This will lead to an increase in the End-to-End efficiency of the QM.

(4) *Bright EPP sources*. Work is required to further improve narrow-band EPP sources to integrate them with fiber-optic circuits and QMs.

(5) *Improvement of optical and electronic elements*. Scientific and engineering work on improving the operating parameters of basic optical elements (acousto-optical and electro-optical modulators, cavities, optical filters, passive optical elements, etc.).

(6) *Electronics*. It is necessary to develop a scalable electronic complex for control and synchronization of all units of a quantum repeater.

* Author’s note to English.

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